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2-Permutations of lattice vertex operator algebras: Higher rank

Chongying Dong [a,1], Feng Xu [b,c,2], Nina Yu [d,*]

[a] *Department of Mathematics, University of California at Santa Cruz, CA 95064, USA*
[b] *Department of Mathematics, Foshan University, Guangdong 528000, China*
[c] *University of California at Riverside, CA 92521, USA*
[d] *School of Mathematical Sciences, Xiamen University, Fujian 361005, China*

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ABSTRACT

This paper is a continuation of our investigation on 2-permutation of lattice vertex operator algebras. In particular, the quantum dimensions of irreducible modules and the fusion rules are determined. If the rank of the lattice is one, these results have been obtained previously in [27].

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1. Introduction

Let V be a vertex operator algebra and n a fixed positive integer and consider the tensor product vertex operator algebra $V^{\otimes n}$ [28]. Then the symmetric group S_n acts naturally on $V^{\otimes n}$ as automorphisms. The permutation orbifold theory has been studied

* Corresponding author.
E-mail address: ninayu@xmu.edu.cn (N. Yu).
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extensively in physics [34,29,13,8]. Conformal nets approach to permutation orbifolds have been given in [33]. Twisted sectors of permutation orbifolds of tensor products of an arbitrary vertex operator algebra have been constructed in [9]. The C_2-cofiniteness of permutation orbifolds and general cyclic orbifolds have been studied in [3,4,38]. But the representation theory such as rationality, classification of irreducible modules, and fusion rules for the fixed point vertex operator algebra $(V^{\otimes n})^G$ for any n and any subgroup G of S_n have not been investigated much.

As a starting point, we studied representations of 2-permutation orbifold model of rank one lattice vertex operator algebras in [27]. In this paper, we complete the study of 2-permutation orbifold model of lattice vertex operator algebras V_L for any positive definite even lattice L. Similar to rank one case, the permutation orbifold model $(V_L \otimes V_L)^{\mathbb{Z}_2}$ can be realized as a simple current extension of the rational vertex operator algebra $V_{\sqrt{2}L} \otimes V^+_{\sqrt{2}L}$. It follows from [41,32] that $(V_L \otimes V_L)^{\mathbb{Z}_2}$ is rational. According to [11,25], every irreducible $(V_L \otimes V_L)^{\mathbb{Z}_2}$-module occurs in an irreducible g-twisted $V_L \otimes V_L$-module. So the classification of irreducible $(V_L \otimes V_L)^{\mathbb{Z}_2}$-modules is known. But this classification result does not suggest how to compute the fusion rules among the irreducible modules. The main idea is to use the general theory of simple current extension of a rational vertex operator algebra and representations of V_L and V_L^+ to study the representations of $(V_L \otimes V_L)^{\mathbb{Z}_2}$. We decompose each irreducible $V_L \otimes V_L$-module into a direct sum of irreducible $(V_L \otimes V_L)^{\mathbb{Z}_2}$-modules by using the fusion rules for both vertex operator algebras $V_{\sqrt{2}L}$ and $V^+_{\sqrt{2}L}$ [18,1,7]. This decomposition is crucial in computing the fusion rules. We emphasize that the theory of quantum dimensions introduced and studied in [17,25] plays an essential role in computing the fusion rules. It is not clear to us how to achieve this without using the quantum dimensions. The fusion rules in conformal nets for any 2-permutation models were computed by using the S-matrix [33].

We should mention that the constructions of g-twisted modules for lattice vertex operator algebra V_L where g is automorphism of finite order induced from an isometry of L were already given in [30,31,35,19]. In the case g is of order 2, the irreducible modules of $V_L^{\langle g \rangle}$ have been classified recently in [11]. An equivalence of two constructions of permutation-twisted modules for lattice vertex operator algebras in [30,35,9] was given in [10].

The paper is organized as follows: §2 and §3 are preliminaries on the vertex operator algebras theory. In these sections we give some basic notions that appear in this paper and recall the constructions of the lattice type vertex operator algebras V_L and V_L^+ and their (twisted) modules. In §4 we study $(V_L \otimes V_L)^{\mathbb{Z}_2}$, the 2-cyclic permutation orbifold models for rank d lattice vertex operator algebras. In particular, we decompose each irreducible $V_L \otimes V_L$-module into a direct sum of irreducible $(V_L \otimes V_L)^{\mathbb{Z}_2}$-modules. The quantum dimensions of all irreducible modules of $(V_L \otimes V_L)^{\mathbb{Z}_2}$ are obtained explicitly in §5. Finally, we apply results from the previous sections to determine all the fusion products in §6.

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