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Ring theoretical properties of affine cellular algebras

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A B S T R A C T

As a generalisation of Graham and Lehrer's cellular algebras, *affine cellular algebras* have been introduced in [12] in order to treat affine Hecke algebras of type A and affine versions of diagram algebras like affine Temperley–Lieb algebras in a unifying fashion. Affine cellular algebras include Kleshchev's graded quasihereditary algebras, Khovanov–Lauda–Rouquier algebras and various other classes of algebras. In this paper we will study ring theoretical properties of affine cellular algebras. We show that any affine cellular algebra A satisfies a polynomial identity. Furthermore, we show that A can be embedded into its asymptotic algebra if the occurring commutative affine k-algebras B_j are reduced and the determinants of the swich matrices are non-zero divisors. As a consequence, we show that the Gelfand–Kirillov dimension of A is less than or equal to the largest Krull dimension of the algebras B_j and that equality holds, in case all affine cell ideals are idempotent or if the Krull dimension of the algebras B_j is less than or equal to 1. Special emphasis is given to the question when an affine cell ideal is idempotent, generated by an idempotent or finitely generated.

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1. Introduction

Affine cellular algebras have been introduced in [12] as a generalisation of Graham and Lehrer's cellular algebras. Affine Hecke algebras of type A and affine versions of diagram algebras like affine Temperley–Lieb algebras are examples of affine cellular algebras. In this paper we will study ring theoretical properties of affine cellular algebras. Recall that an affine cellular algebra A over a commutative Noetherian ring k has a chain of ideals $0 = J_{-1} \subset J_0 \subset J_1 \subset \cdots \subset J_n = A$, such that J_j/J_{j-1} is an affine cell ideal of A/J_{j-1} and as such is isomorphic, as an A/J_{j-1}-bimodule, to a generalised matrix ring $\widetilde{M_{m_j}(B_j)}$ over some affine commutative k-algebra whose multiplication is deformed by a swich matrix $\psi_j \in M_{m_j}(B_j)$. Since the publication of [12], several classes of algebras, like the Khovanov–Lauda–Rouquier algebras, Kleshchev's graded quasihereditary algebras, the affine Birman–Murakami–Wenzl algebras, affine Brauer algebras, affine q-Schur algebras and BLN-algebras were shown to be affine cellular (see [4–6,8–11,16]). These classes of algebras are subclasses of affine cellular algebras and contain other interesting examples, like Kato's geometric extension algebras (see [11, 10.2]). Although it was shown that many algebras are affine cellular, their ring theoretical structure has not been studied in much detail apart from [12].

Affine cellular algebras are built up by affine cell ideals, which will be studied first. For any element $\psi \in M_n(B)$ of the $n \times n$-matrix ring $M_n(B)$ over a commutative affine k-algebra B, with k a commutative Noetherian ring, the generalised matrix ring is the associative (possibly non-unital) ring $\widetilde{M_n(B)}$ which is, as a k-module, equal to $M_n(B)$ but whose multiplication is deformed by setting $a * b := a\psi b$, for elements $a, b \in \widetilde{M_n(B)}$. An affine cell ideal J of an algebra A is isomorphic as a ring to a generalised matrix ring $\widetilde{M_n(B)}$. By [12, Theorem 4.1], idempotent affine cell ideals are important for the understanding of the representation theory of affine cellular algebras. We show in Theorem 3.3 and Proposition 3.5 that

(1) J is an idempotent ideal if and only if B is generated (as an ideal) by the entries of ψ. In this case J is a finitely generated left and right ideal of A.
(2) J is generated by an idempotent as left ideal if and only if J is a principal left ideal of A if and only if $\det(\psi)$ is invertible in B. In this case, the idempotent must be central and A decomposes as the ring direct product $A \simeq A/J \times M_n(B)$.
(3) $\mathrm{End}(_AJ) \simeq M_n(B)$ if J is idempotent or if J contains an element that is a central non-zero divisor in J. The latter case is fulfilled in case $\det(\psi)$ is a non-zero divisor in B.

Statement (1) gives an alternative description of idempotent affine cell ideals compared with the equivalent conditions found in [12, Theorem 4.1]. Statement (2) clarifies the relation between an affine cell ideal being generated by an idempotent and being an idempotent ideal. Statement (3) has been shown in [12, Theorem 4.3] for idempotent

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