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Spectral analysis of non-local Schrödinger operators.

Yu. Kondratiev [*], S. Molchanov [†], B. Vainberg [‡]

Abstract

We study spectral properties of convolution operators $\mathcal{L}$ and their perturbations $H=\mathcal{L}+v(x)$ by compactly supported potentials. Results are applied to determine the front propagation of a population density governed by operator H with a compactly supported initial density provided that H has positive eigenvalues. If there is no positive spectrum, then the stabilization of the population density is proved.

Key words: random walk, spectrum, front propagation, population, stabilization.
MSC classification: 47A10, 60J85, 47D06

1 Introduction

In this paper we study the spectral properties of the Hamiltonian H in $L^2(R^d)$ given by

$$H=\mathcal{L}+v(x), \quad \mathcal{L}\psi(x)=\chi\int_{y\in R^d}(\psi(x+y)-\psi(x))a(y)dy, \quad x\in R^d, \tag{1}$$

where

$$a(y)=a(-y), \quad a\geq 0, \quad \int_{R^d}a(y)dy=1, \tag{2}$$

and $v(x)\geq 0$ is continuous and compactly supported.

The operator $\mathcal{L}$ is the generator of a symmetric random walk on R^d with the intensity of jumps equal to $\chi>0$. Function $a(y)$ is the density of transition from x to $x+y$ at the moment of the jump. Operators H of the form (1) appear in many applications, such as models of population dynamics that include the KPP type processes (where the offspring start at the location of the parent particle [9], [4], [5], [6]) and contact processes

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