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Trace formulas for Wiener–Hopf operators with applications to entropies of free fermionic equilibrium states

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A B S T R A C T

We consider non-smooth functions of (truncated) Wiener–Hopf type operators on the Hilbert space $\mathsf{L}^2(\mathbb{R}^d)$. Our main results are uniform estimates for trace norms ($d \geq 1$) and quasiclassical asymptotic formulas for traces of the resulting operators ($d = 1$). Here, we follow Harold Widom's seminal ideas, who proved such formulas for smooth functions decades ago. The extension to non-smooth functions and the uniformity of the estimates in various (physical) parameters rest on recent advances by one of the authors (AVS). We use our results to obtain the large-scale behaviour of the local entropy and the spatially bipartite entanglement entropy (EE) of thermal equilibrium states of non-interacting fermions in position space $\mathbb{R}^d$ ($d \geq 1$) at positive temperature, $T > 0$. In particular, our definition of the thermal EE leads to estimates that are simultaneously sharp for small T and large scaling parameter $\alpha > 0$ provided that the product $T\alpha$ remains bounded from below. Here α is the reciprocal quasiclassical parameter. For $d = 1$ we obtain for the thermal EE an asymptotic formula which is consistent with the large-scale

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behaviour of the ground-state EE (at $T = 0$), previously established by the authors for $d \geq 1$.

Contents

1. Introduction

The present paper is devoted to the study of (bounded, self-adjoint) operators of the form

$$W_\alpha := W_\alpha(a; \Lambda) := \chi_\Lambda \operatorname{Op}_\alpha(a)\chi_\Lambda, \ \alpha > 0, \tag{1.1}$$

on $\mathsf{L}^2(\mathbb{R}^d)$, $d \geq 1$, where χ_Λ is the indicator function of a set $\Lambda \subset \mathbb{R}^d$. The parameter $1/\alpha$ can be interpreted as a quasiclassical parameter that tends to zero in our asymptotic results. The notation $\operatorname{Op}_\alpha(a)$ stands for the α-pseudo-differential operator with symbol $a = a(\boldsymbol{\xi})$, which acts on Schwartz functions u on $\mathbb{R}^d$ as

$$\big(\operatorname{Op}_\alpha(a)u\big)(\mathbf{x}) := \frac{\alpha^d}{(2\pi)^{\frac{d}{2}}} \iint e^{i\alpha\boldsymbol{\xi}\cdot(\mathbf{x}-\mathbf{y})} a(\boldsymbol{\xi})u(\mathbf{y})d\mathbf{y}d\boldsymbol{\xi}, \quad \mathbf{x} \in \mathbb{R}^d.$$

Integrals without indication of the integration domain always mean integration over $\mathbb{R}^d$ with the value of d which is clear from the context. More general symbols, depending on both variables $\mathbf{x}$ and $\boldsymbol{\xi}$, or operators with matrix-valued symbols can be also treated, but we limit our attention only to $\boldsymbol{\xi}$-dependent symbols. We call the operator (1.1)