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Essential self-adjointness of powers of first-order differential operators on non-compact manifolds with low-regularity metrics

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We consider first-order differential operators with locally bounded measurable coefficients on vector bundles with measurable coefficient metrics. Under a mild set of assumptions, we demonstrate the equivalence between the essential self-adjointness of such operators to a negligible boundary property. When the operator possesses higher regularity coefficients, we show that higher powers are essentially self-adjoint if and only if this condition is satisfied. In the case that the low-regularity Riemannian metric induces a complete length space, we demonstrate essential self-adjointness of the operator and its higher powers up to the regularity of its coefficients. We also present applications to Dirac operators on Dirac bundles when the metric is non-smooth.

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Contents

1. Introduction

The problem of determining essential self-adjointness of smooth coefficient first-order differential operators, as well as their powers, is an important and well studied topic. This paper considers similar problems but in the context of non-smooth coefficients. More precisely, we consider first-order symmetric differential operators D, as well as their powers, on smooth vector bundles $\mathcal{V}$, over smooth, noncompact manifolds $\mathcal{M}$. We allow for the coefficients of the operator as well as the metrics on the bundle and the manifold to be non-smooth. Our primary focus is to understand the relationship between the regularity of the coefficients of D and the essential self-adjointness of powers of D.

One of the primary motivations for studying the essential self-adjointness of a differential operator D comes from the fact that it allows one to build a functional calculus (of Borel functions) for the closure of that operator. Such a functional calculus can then, for instance, be used to build a heat operator, $e^{-t\mathrm{D}}$ for $t > 0$, and a Schrodinger operator, $e^{\imath t\mathrm{D}}$ for $t \in \mathbb{R}$. These in turn can then be used to solve the heat equation $u_t + \mathrm{D}u = 0$, and the Schrodinger equation $u_t + \imath \mathrm{D}u = 0$ respectively. It is in the construction of such solutions to differential equations that makes essential self-adjointness an indispensable property.

There is a plethora of historical literature surrounding this subject, and therefore, we confine ourselves to presenting only the relevant references to our work. From our point of view, it was Gaffney who in [16] made a first significant contribution by establishing the essential self-adjointness of the Hodge-Laplacian $(\mathrm{d}\delta + \delta\mathrm{d})$ under a so-called *negligi-*