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Nonlinear damped partial differential equations and their uniform discretizations

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ABSTRACT

We establish sharp energy decay rates for a large class of nonlinearly first-order damped systems, and we design discretization schemes that inherit of the same energy decay rates, uniformly with respect to the space and/or time discretization parameters, by adding appropriate numerical viscosity terms. Our main arguments use the optimal-weight convexity method and uniform observability inequalities with respect to the discretization parameters. We establish our results, first in the continuous setting, then for space semi-discrete models, and then for time semi-discrete models. The full discretization is then inferred from the previous results, by adapting the ideas to deal with linear systems.

Our results cover, for instance, the Schrödinger equation with nonlinear damping, the nonlinear wave equation, the

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nonlinear plate equation, the nonlinear transport equation, as well as certain classes of equations with nonlocal terms.

Contents

1. Introduction

Let X be a Hilbert space. Throughout the paper, we denote by $\|\cdot\|_X$ the norm on X and by $\langle\cdot,\cdot\rangle_X$ the corresponding scalar product. Let $A : D(A) \to X$ be a densely defined skew-adjoint operator, and let $B : X \to X$ be a nontrivial bounded selfadjoint nonnegative operator. Let $F : X \to X$ be a (nonlinear) mapping, assumed to be Lipschitz continuous on bounded subsets of X. We consider the differential system

$$u'(t) + Au(t) + BF(u(t)) = 0. \tag{1}$$

If $F = 0$ then the system (1) is conservative, and for every $u_0 \in D(A)$, there exists a unique solution $u(\cdot) \in C^0(0,+\infty; D(A)) \cap C^1(0,+\infty; X)$ such that $u(0) = u_0$, which satisfies moreover $\|u(t)\|_X = \|u(0)\|_X$, for every $t \geq 0$.

If $F \neq 0$ then the system (1) is expected to be dissipative if the nonlinearity F has "the good sign". Defining the energy of a solution u of (1) by

$$E_u(t) = \frac{1}{2}\|u(t)\|_X^2, \tag{2}$$

we have, as long as the solution is well defined,