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Characterization of inclusion relations between Wiener amalgam and some classical spaces ☆

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In this paper, we establish the sharp conditions for the inclusion relations between Besov spaces $B_{p,q}$ and Wiener amalgam spaces $W^s_{p,q}$. We also obtain the optimal inclusion relations between local hardy spaces h^p and Wiener amalgam spaces $W^s_{p,q}$, which completely improve and extend the main results obtained by Cunanana, Kobayashib and Sugimotoa in [3]. In addition, we establish some mild characterizations of inclusion relations between Triebel–Lizorkin and Wiener amalgam spaces, which relates some modern inequalities to classical inequalities.

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1. Introduction

Wiener amalgam spaces are a class of function spaces which amalgamates local properties with global properties. Specific cases of these spaces was first introduced by Norbert Wiener in [30–32]. In 1980s, H.G. Feichtinger developed a far-reaching generalization of amalgam spaces, which allows a wide range of Banach spaces to serve as local or global components. Feichtinger used $W(B,C)$ to denote the Wiener-type spaces, which B and C are served as the local and global component respectively. Here, we consider a limited case, namely, the Wiener amalgam spaces $W^s_{p,q}$, which can be re-expressed as $W(\mathscr{F}^{-1}L^s_q, L_p)$ by the notation of Feichtinger. In addition, modulation space $M^s_{p,q}$ can be re-expressed by $\mathscr{F}^{-1}W(\mathscr{F}L_p, L^s_q)$.

By the frequency-uniform localization techniques, the modulation space can be viewed as a Besov-type space associated with a uniform decomposition (see Triebel [24], Wang–Huang [28]). Like the modulation space, Wiener amalgam space $W^s_{p,q}$ can be viewed as a Triebel–Lizorkin-type space corresponding to a uniform decomposition, we refer the readers to [24] for this topic. Due to the difference between uniform and dyadic decompositions, Wiener amalgam space $W^s_{p,q}$ has many properties different from the corresponding dyadic space. For instance, the uniform multiplier $e^{i|D|^\beta}(0 < \beta < 1)$ is unbounded in the classical Lebesgue spaces L_p but bounded on $W^s_{p,q}$ for $1 \leqslant p, q \leqslant \infty$, $s \in \mathbb{R}$, one can see [1,4] for more details. Thus, it is interesting to compare the Wiener amalgam spaces with the classical function spaces constructed by dyadic decompositions. One basic but important problem is how to characterize the inclusion relations between theses two kinds of spaces. A lot of attention have been paid to this topic, for example, one can see [20,21,28] for the inclusion relations between modulation and Besov spaces, [11,13,23] for the more general inclusion relations under the frame of α-modulation space, [16] for the inclusion relations between local Hardy spaces h_p and modulation spaces, and [17] for the inclusion relation between Sobolev spaces and modulation spaces, *etc.* In particular, Cunanana, Kobayashib and Sugimotoa [3] recently gave some necessary and sufficient conditions for the inclusion relations between $W^s_{p,q}$ and L_p in the range of $1 \leqslant p, q \leqslant \infty$, they also give a corollary for the inclusion relations between $W^s_{p,q}$ and $B_{p,q}$ (the Besov space $B^0_{p,q}$, see (2.16) for the definition). We will recall the main results in [3] after introducing the following notation.

Denote $\alpha(p,q) = \max\{0, n(1-1/p-1/q), n(1/2-1/q)\}$, $\beta(p,q) = \min\{0, n(1-1/p-1/q), n(1/2-1/q)\}$, that is,

$$\alpha(p,q) = \begin{cases} 0, & \text{if } (1/p,1/q) \in A_1 : \ 1/q \geqslant \max\{1-1/p, 1/2\}, \\ n(1-1/p-1/q), & \text{if } (1/p,1/q) \in A_2 : \ 1/p \leqslant \min\{1-1/q, 1/2\}, \\ n(1/2-1/q), & \text{if } (1/p,1/q) \in A_3 : \ 1/q \leqslant 1/2 \leqslant 1/p; \end{cases}$$