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Boundary regularity and sufficient conditions for strong local minimizers

Judith Campos Cordero [1]

Institut für Mathematik, Universität Augsburg, Universitätsstrasse 14, 86159, Augsburg, Germany

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A B S T R A C T

In this paper we present a new proof of the sufficiency theorem for strong local minimizers concerning C^1-extremals at which the second variation is strictly positive. The results are presented in the quasiconvex setting, in accordance with the original statement by Grabovsky and Mengesha [31]. The strategy that we follow relies on a Decomposition Theorem that allows to split a sequence of variations into its oscillating and its concentrating parts, as well as on a sufficiency result according to which smooth extremals are spatially-local minimizers. Furthermore, we prove partial regularity up to the boundary for strong local minimizers in the non-homogeneous case and a full regularity result for Lipschitz extremals with gradient of vanishing mean oscillation. As a consequence, we also establish a sufficiency result for this class of extremals. The regularity results are established via a blow-up argument.

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E-mail address: judith.campos@xanum.uam.mx.

[1] *Present address:* Universidad Autónoma Metropolitana – Iztapalapa, Departamento de Matemáticas, Av. San Rafael Atlixco No. 186 Col. Vicentina, C.P. 09340, Ciudad de México, México.

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1. Introduction

It is well known that minimizers of variational integrals satisfy the Euler–Lagrange equations and the nonnegativity of the second variation. On the other hand, for non-convex integrands we are faced with the possibility of encountering local minimizers that are not globally minimizing the functional. Hence, an underlying problem in the Calculus of Variations has been to find sufficient conditions guaranteeing that an extremal at which the second variation is positive is in fact a strong local minimizer.

This problem was solved by Weierstrass in the 19th century for the case in which the admissible scalar functions have one single independent variable. Levi then provided a proof with a different method, which does not use the field theory originated from Weierstrass' work [44]. The sufficiency result was later generalized by Hestenes for functions of several variables [34].

On the other hand, Meyers showed that the notion of quasiconvexity developed by Morrey [48] is in a suitable sense a necessary condition for strong local minimizers [45]. Furthermore, Ball and Marsden established the concept of *quasiconvexity at the free boundary* in [6], where this was also shown to be a necessary condition for strong local minimizers satisfying mixed boundary conditions, that is, by allowing the minimizers to take free values on part of the boundary of their domain.

The importance of obtaining an adequate set of sufficient conditions for strong local minima in the vectorial case was highly motivated by applications coming from materials science. Ball foresaw that the natural way to extend Weierstrass condition to the vectorial case are the notions of quasiconvexity at the interior and at the free boundary. Furthermore, in [5, Section 6.2] it is conjectured that if a solution to the weak Euler–Lagrange equation is sufficiently smooth, then the strict positivity of the second variation together with suitable versions of the quasiconvexity in the interior and at the boundary should guarantee that the extremal furnishes a strong local minimizer.

Further generalizations regarding Weierstrass problem were obtained by Taheri in [56], where Hestenes' strategy is extended to the case of L^p-local minimizers.