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Strong solutions to the 3D primitive equations with only horizontal dissipation: Near H^1 initial data

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ABSTRACT

In this paper, we consider the initial–boundary value problem of the three-dimensional primitive equations for oceanic and atmospheric dynamics with only horizontal viscosity and horizontal diffusivity. We establish the local, in time, well-posedness of strong solutions, for any initial data $(v_0, T_0) \in H^1$, by using the local, in space, type energy estimate. We also establish the global well-posedness of strong solutions for this system, with any initial data $(v_0, T_0) \in H^1 \cap L^\infty$, such that $\partial_z v_0 \in L^m$, for some $m \in (2, \infty)$, by using the logarithmic type anisotropic Sobolev inequality and a logarithmic type Gronwall inequality. This paper improves the previous results obtained in Cao et al. (2016) [10], where the initial data (v_0, T_0) was assumed to have H^2 regularity.

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1. Introduction

In the context of the large-scale oceanic and atmospheric dynamics, an important feature is that the vertical scale (10–20 kilometers) is much smaller than the horizontal scales (several thousands of kilometers), and therefore, the aspect ratio, i.e. the ratio of the depth (or hight) to the horizontal width, is very small. Due to this fact, by the scale analysis (see, e.g., Pedlosky [41]), or taking the small aspect ratio limit to the Navier–Stokes equations (see Azérad–Guillén [1] and Li–Titi [33,35] for the mathematical justification of this limit), one obtains the primitive equations. The primitive equations form a fundamental block in models for planetary oceanic and atmospheric dynamics, and are wildly used in the models of the weather prediction, see, e.g., the books by Haltiner–Williams [21], Lewandowski [29], Majda [40], Pedlosky [41], Vallis [46], Washington–Parkinson [47] and Zeng [49]. Moreover, in the oceanic and atmospheric dynamics, due to the strong horizontal turbulent mixing, the horizontal viscosity and diffusivity are much stronger than the vertical viscosity and diffusivity, respectively.

In this paper, we consider the following version of the primitive equations for oceanic and atmospheric dynamics, which have only horizontal dissipation, i.e. with only horizontal viscosity and horizontal diffusivity

$$\partial_t v + (v \cdot \nabla_H) v + w \partial_z v + \nabla_H p - \Delta_H v + f_0 k \times v = 0, \tag{1.1}$$

$$\partial_z p + T = 0, \tag{1.2}$$

$$\nabla_H \cdot v + \partial_z w = 0, \tag{1.3}$$

$$\partial_t T + (v \cdot \nabla_H) T + w \partial_z T - \Delta_H T = 0, \tag{1.4}$$

where the horizontal velocity $v = (v^1, v^2)$, the vertical velocity w, the temperature T and the pressure p are the unknowns, and f_0 is the Coriolis parameter. The notations $\nabla_H = (\partial_x, \partial_y)$ and $\Delta_H = \partial_x^2 + \partial_y^2$ are the horizontal gradient and the horizontal Laplacian, respectively. Notably, the above system has been first studied by the authors in [10], where the global existence of strong solutions was established, for arbitrary initial data with H^2 regularity; the aim of the present paper is to relax the conditions on the initial data, without losing the global well-posedness of strong solutions.

The mathematical studies of the primitive equations were started by Lions–Temam–Wang [37–39] in the 1990s, where among other issues, global existence of weak solutions was established; however, the uniqueness of weak solutions is still an open question, even for the two-dimensional case. Note that this is different from the incompressible Navier–Stokes equations, as it is well-known that the weak solutions to the two-dimensional incompressible Navier–Stokes equations are unique, see, e.g., Constantin–Foias [15], Ladyzhenskaya [28], Temam [45] and more recently Bardos et al. [2] for the uniqueness of weak solutions, within the class of three-dimensional Leray–Hopf weak solutions, with initial data that are functions of only two spatial variables. However, we would like to point out that, though the general uniqueness of weak solutions to the primitive