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Partial actions and subshifts ☆

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ABSTRACT

Given a finite alphabet Λ, and a not necessarily finite type subshift $X \subseteq \Lambda^\infty$, we introduce a partial action of the free group $\mathbb{F}(\Lambda)$ on a certain compactification Ω_X of X, which we call the spectral partial action.

The space Ω_X has already appeared in many papers in the subject, arising as the spectrum of a commutative C*-algebra usually denoted by $\mathcal{D}_X$. A good understanding of $\mathcal{D}_X$ is crucial for the study of C*-algebras related to subshifts, and since the descriptions given of Ω_X in the literature are often somewhat terse and obscure, one of our main goals is to present a sensible model for it which allows for a detailed study of its structure, as well as of the spectral partial action, from various points of view, including topological freeness and minimality.

We then apply our results to study certain C*-algebras associated to X, introduced by Matsumoto and Carlsen. Thus the spectral partial action permits us to endow the Carlsen–Matsumoto C*-algebra $\mathcal{O}_X$ with a partial crossed product structure. We combine this with our characterization of the dynamical properties of the spectral partial action, in order to treat the problem of simplicity of $\mathcal{O}_X$, considered earlier by several authors. As a new advance, we are able to give necessary and sufficient conditions for $\mathcal{O}_X$ to be simple, without imposing any restriction on X, and this is done in terms of transparent "graphical" properties of X. As a by-product of our partial action approach, we easily recover

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some known facts on $\mathcal{O}_X$, putting them more in line with mainstream techniques used to treat similar C*-algebras.

1. Introduction

The theory of C*-algebras associated to subshifts has a long and exciting history, having been initiated by Matsumoto in [26], later receiving invaluable contributions from many other authors, notably Carlsen, Silvestrov, and Thomsen [8–11,37]. Accounts of this history may be found in [9] and [7], to which the interested reader is referred.

For now it suffices to say that, given a finite alphabet Λ, and a closed subset $X \subseteq \Lambda^{\mathbb{N}}$ which is invariant under the left shift

$$S : \Lambda^{\mathbb{N}} \to \Lambda^{\mathbb{N}},$$

in which case one says that the pair (X, S) is a (one sided) *subshift*, Matsumoto initiated a study of certain C*-algebras associated to X, whose algebraic properties reflect certain important dynamical properties of the subshift itself, and whose K-theory groups provide new invariants for subshifts.

The extensive literature in this field (see for instance the list of references in [8]) contains a lot of information about the structure of these algebras, such as faithful representations, nuclearity, simplicity, computation of K-theory groups and a lot more. It is therefore a perilous task to attempt to add anything else to the wealth of results currently available, a task we hope to be undertaking in a responsible manner.

The motivation that brought us to revisit the theory of C*-algebras associated to subshifts, and the justification for writing the present paper, is twofold: firstly we are able to offer a sensible description of a certain topological space, which we will denote by Ω_X (for the cognoscenti, we are referring to the spectrum of the ill-fated commutative algebra $\mathcal{D}_X$ appearing in most papers on the subject), and which has evaded all attempts at analysis, except maybe for some somewhat obscure projective limit descriptions given in [27] and [6, Section 2.1]. More recently Starling [36] was able to describe this space as the tight spectrum [17, Definition 12.8] of the idempotent semilattice of a certain inverse semigroup associated to X. See also [34] and [35].

Secondly we will introduce a partial action of the free group $\mathbb{F} = \mathbb{F}(\Lambda)$ on Ω_X, called the *spectral partial action*, whose associated crossed product is the Carlsen–Matsumoto C*-algebra $\mathcal{O}_X$. Given that description, we may recover many known results for $\mathcal{O}_X$, as well as give the first necessary and sufficient condition for simplicity which applies for all subshifts, including those where the shift map is not surjective.

The method we adopt in our study of the space Ω_X is essentially the same one used by the second named author and M. Laca in the analysis of Cuntz–Krieger algebras for infinite matrices [20], the crucial insight being the introduction of a partial action of the