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Multiplicity bound of singular spectrum for higher rank Anderson models

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ABSTRACT

In this work, we prove a bound on the multiplicity of the singular spectrum for a certain class of Anderson Hamiltonians. The operator in consideration is the form $H^\omega = \Delta + \sum_{n\in\mathbb{Z}^d} \omega_n P_n$ on the Hilbert space $\ell^2(\mathbb{Z}^d)$, where Δ is discrete laplacian, P_n are projection onto $\ell^2(\{x \in \mathbb{Z}^d : n_i l_i < x_i \leq (n_i + 1)l_i\})$ for some $l_1, \cdots, l_d \in \mathbb{N}$ and $\{\omega_n\}_n$ are i.i.d. real bounded random variables following an absolutely continuous distribution. We prove that the multiplicity of the singular spectrum is bounded above by $2^d - d$ independent of $\{l_i\}_{i=1}^d$. When $l_i+1 \notin 2\mathbb{N}\cup 3\mathbb{N}$ for all i and $gcd(l_i+1, l_j+1) = 1$ for $i \neq j$, the singular spectrum is also simple.

1. Introduction

Random Schrödinger operators and their tight binding version, Anderson model, are well-studied for their spectral properties. Multiplicity of spectrum is an important part to understand the structure of any operator, but it also shows up in the study of spectral

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statistics and localization theory, for example Hislop–Krishna [9], Germinet–Klein [7], Combes–Hislop [6] and Combes–Germinet–Klein [5].

It becomes, therefore, important to know the multiplicity of the spectrum, mainly in the random Schrödinger case. In the Anderson tight binding model, which is the rank one case, Barry Simon [20] showed that any standard basis vector δ_n is cyclic in the pure point region of the spectrum. Other works in the pure point regime are by Klein–Molchanov [14] and Aizenman–Warzel [2]. Jakšić–Last in [10,12] showed that the singular spectrum is almost surely simple in the case of Anderson type Hamiltonians where the rank of the perturbation is one.

But for the higher rank case, spectral simplicity is not always true. In [19] Sadel and Schulz–Baldes worked with a certain family of random Dirac operators and showed non-trivial multiplicity of the spectrum depending on certain parameter defining the model. Though Naboko–Nichols–Stolz [17] showed that the point spectrum is simple for the operator (1.1) given below, in some particular cases.

We consider the higher rank Anderson model as a first step towards any result concerning the multiplicity of the singular spectrum for random Schrödinger operator. Based on the heuristics that non-trivial multiplicity for point spectrum arises from underlying symmetry, and randomness breaks all symmetry almost surely, it can easily be conjectured that the singular spectrum is simple for the family of operators given by (1.1). But here we prove that the singular spectrum is simple for a large family of operator of form (1.1) and in general the multiplicity of singular spectrum is bounded by $2^d - d$.

The Hamiltonian we will work on is like the Anderson tight binding model, except that the perturbations are equal over boxes. The class of operators in consideration are as follows. On the Hilbert space $\ell^2(\mathbb{Z}^d)$, the family of operators in consideration are given by

$$H^\omega = \Delta + \sum_{n\in\mathbb{Z}^d} \omega_n P_n, \tag{1.1}$$

where Δ and $\{P_n\}_{n\in\mathbb{Z}^d}$ are

$$(\Delta u)(x) = \sum_{i=1}^{d} u(x+e_i) + u(x-e_i),$$

and

$$(P_n u)(x) = \begin{cases} u(x) & n_i l_i < x_i \le (n_i+1) l_i \ \forall i \\ 0 & \text{otherwise} \end{cases},$$

where $\{e_i\}_{i=1}^d$ is the standard generator of the group $\mathbb{Z}^d$ and $l_1, \cdots, l_d \in \mathbb{N}$. The sequence $\{\omega_n\}_{n\in\mathbb{Z}^d}$ are i.i.d. real bounded random variables following absolutely continuous distribution μ. The figure gives a representation for $d = 2$.