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Reflexivity of non-commutative Hardy algebras

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ABSTRACT

Let $H^\infty(E)$ be a non-commutative Hardy algebra associated with a W^*-correspondence E. These algebras were introduced in 2004 by Muhly and Solel, and generalize the classical Hardy algebra of the unit disc $H^\infty(\mathbb{D})$. As a special case one obtains also the algebra $\mathcal{F}_d^\infty$ of Popescu, which is $H^\infty(\mathbb{C}^d)$ in our setting.

In this paper we view the algebra $H^\infty(E)$ as acting on a Hilbert space via an induced representation. We write it $\rho_\pi(H^\infty(E))$ and we study the reflexivity of $\rho_\pi(H^\infty(E))$. This question was studied by Arias and Popescu in the context of the algebra $\mathcal{F}_d^\infty$, and by other authors in several other special cases. As it will be clear from our work, the extension to the case of a general W^*-correspondence E over a general W^*-algebra M requires new techniques and approach.

We obtain some partial results in the general case and we turn to the case of a correspondence over a factor. Under some additional assumptions on the representation $\pi : M \to B(H)$ we show that $\rho_\pi(H^\infty(E))$ is reflexive. Then we apply these results to analytic crossed products $\rho_\pi(H^\infty({}_\alpha M))$ and obtain their reflexivity for any automorphism $\alpha \in \mathrm{Aut}(M)$ whenever M is a factor. Finally, we show also the reflexivity of the compression of the Hardy algebra to a suitable coinvariant subspace $\mathfrak{M}$, which may be thought of as a generalized symmetric Fock space.

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1. Introduction

In this paper we consider the question of reflexivity of the non-commutative Hardy algebras $\rho_\pi(H^\infty(E))$. These algebras were introduced by Muhly and Solel in [21,23], and generalize a wide class of nonselfadjoint operator algebras, in particular they can be viewed as a far reaching generalization of the classical Hardy algebra $H^\infty(\mathbb{D})$.

The notion of reflexivity is one of the central notions in the theory of operator algebras. Let $\mathcal{A}$ be any algebra acting on a Hilbert space H. Write $\mathrm{Lat}(\mathcal{A})$ for the lattice of all $\mathcal{A}$-invariant closed subspaces in H. Thus,

$$\mathrm{Lat}(\mathcal{A}) := \{\mathcal{M} \subseteq H : A\mathcal{M} \subseteq \mathcal{M},\ \ \forall A \in \mathcal{A}\}.$$

For any lattice $\mathcal{L}$ of closed subspaces in H let $\mathrm{Alg}(\mathcal{L})$ be the operator algebra

$$\mathrm{Alg}(\mathcal{L}) = \{A \in B(H) : A\mathcal{M} \subseteq \mathcal{M},\ \ \forall \mathcal{M} \in \mathcal{L}\}.$$

Clearly, for any operator algebra $\mathcal{A}$ we have $\mathcal{A} \subseteq \mathrm{AlgLat}(\mathcal{A})$. An operator algebra $\mathcal{A}$ is said to be *reflexive* if $\mathcal{A} = \mathrm{AlgLat}(\mathcal{A})$. Thus, the algebra $\mathcal{A}$ is reflexive if it is defined by its invariant subspaces. It is clear also that a reflexive algebra $\mathcal{A}$ must be a unital WOT-closed, hence ultraweakly closed, subalgebra in $B(H)$. The property to be reflexive depends on the representation $\rho : \mathcal{A} \to B(H)$ of $\mathcal{A}$ on a Hilbert space H, i.e. it is a spatial property of the algebra.

Every von Neumann algebra is reflexive. This fact is equivalent to the bicommutant theorem. Thus, the question of reflexivity is interesting only for nonselfadjoint operator algebras.

A single operator T is called reflexive if the WOT-closed algebra, generated by T and the identity I, is reflexive. We denote this algebra by $W(T)$. In [35] Sarason proved that the unilateral shift S is reflexive. In this case $W(S)$ is the algebra of analytic Toeplitz operators, and the weak topology and the ultraweak topology coincide when restricted to $W(S)$. Thus the Hardy algebra of the unit disc in complex plane is reflexive (we shall see later that this algebra is $\rho_\pi(H^\infty(\mathbb{C}))$ in our setting).

The following simple example shows that not every WOT-closed algebra is reflexive. Let $\mathcal{A}$ be an algebra of all 2×2 matrices over $\mathbb{C}$ of the form $\begin{pmatrix} a & b \\ 0 & a \end{pmatrix}$, for $a, b \in \mathbb{C}$. Clearly, $\mathcal{A}$ is WOT closed. But it is easy to see that

$$\mathrm{AlgLat}(\mathcal{A}) = \{\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} :\ a, b, c \in \mathbb{C}\}.$$

Thus, $\mathcal{A}$ is not reflexive. The notion of reflexivity was introduced first by Radjavi and Rosenthal in [31], and the terminology was suggested by Halmos, [11]. Since the work [31] of Radjavi and Rosenthal appeared, the subject of reflexivity and its generalization has

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