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Normalizations of Eisenstein integrals for reductive symmetric spaces

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ABSTRACT

We construct minimal Eisenstein integrals for a reductive symmetric space G/H as matrix coefficients of the minimal principal series of G. The Eisenstein integrals thus obtained include those from the σ-minimal principal series. In addition, we obtain related Eisenstein integrals, but with different normalizations. Specialized to the case of the group, this wider class includes Harish-Chandra's minimal Eisenstein integrals.

Contents

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0. Introduction

Eisenstein integrals play a fundamental role in harmonic analysis on reductive symmetric spaces of the form $X = G/H$; here G is assumed to be a real reductive group of the Harish-Chandra class, and H an (essentially connected) open subgroup of the group G^σ of an involution σ of G. The notion of Eisenstein integral goes back to Harish-Chandra, who used it to describe the contribution of generalized principal series to the Plancherel decomposition of a real reductive group $`G$. In this setting an Eisenstein integral is essentially a matrix coefficient of an induced representation of the form $\mathrm{Ind}_{`P}^{`G}(`\omega)$, with $`P$ a proper parabolic subgroup of $`G$ and $`\omega$ a suitable representation of $`P$.

For general symmetric spaces G/H, the notion of Eisenstein integral was introduced in [6] for minimal σ-parabolic subgroups of G, i.e., minimal parabolic subgroups of G with the property that $\sigma(P) = \bar{P}$. The notion was later generalized to arbitrary σ-parabolic subgroups in [14,15] and found application in the Plancherel theorem for G/H, see [16] and [12]. In this setting of reductive symmetric spaces, the Eisenstein integrals appear essentially as matrix coefficients of K-finite matrix coefficients with H-fixed distribution vectors.

A group $`G$ of the Harish-Chandra class may be viewed as a homogeneous space for the left times right action of $G = `G \times `G$ on $`G$, and is thus realized as the symmetric space G/H with H the diagonal in G. The definition of Eisenstein integral for the symmetric space G/H yields a matrix coefficient on $`G$ which is closely related to Harish-Chandra's Eisenstein integral, but not equal to it. The two obtained types of Eisenstein integrals differ by a normalization which can be described in terms of intertwining operators, see [8] for details. In the present paper we develop a notion of minimal Eisenstein integrals for reductive symmetric spaces, which cover both the existing notion for symmetric spaces and Harish-Chandra's notion for the group.

An even stronger motivation for the present article lies in the application of its results to a theory of cusp forms for symmetric spaces, initiated by M. Flensted-Jensen. In [7] we use our results on Eisenstein integrals to generalize the results of [2] and [1] to reductive symmetric spaces of σ-split rank one (i.e., $\dim \mathfrak{a}_\mathfrak{q} = 1$).

We will now explain our results in more detail. Let θ be a Cartan involution of G commuting with σ and let K be the associated maximal compact subgroup of G. Let

$$\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{p} = \mathfrak{h} \oplus \mathfrak{q}$$