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Gevrey regularity for Navier–Stokes equations under Lions boundary conditions

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ABSTRACT

The Navier–Stokes system is considered in a compact Riemannian manifold. Gevrey class regularity is proven under Lions boundary conditions: in 2D for the Rectangle, Cylinder, and Hemisphere, and in 3D for the Rectangle. The cases of the 2D Sphere and 2D and 3D Torus are also revisited.

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1. Introduction

Let $\Omega \subset \mathbb{R}^d$, $d \in \{2, 3\}$ be a connected bounded domain located locally on one side of its smooth boundary $\Gamma = \partial\Omega$. The Navier–Stokes system, in $(0, T) \times \Omega$, reads

$$\partial_t u + \langle u \cdot \nabla \rangle u - \nu \Delta u + \nabla p + h = 0, \quad \operatorname{div} u = 0, \quad \mathcal{G}u|_\Gamma = 0, \quad u(0, x) = u_0(x) \tag{1}$$

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where as usual $u = (u_1, \dots, u_d)$ and p, defined for $(t, x_1, \dots, x_d) \in I \times \Omega$, are respectively the unknown velocity field and pressure of the fluid, $\nu > 0$ is the viscosity, the operators ∇ and Δ are respectively the well known gradient and Laplacian in the space variables $(x_1, \dots, x_d)$, $\langle u \cdot \nabla \rangle v$ stands for $(u \cdot \nabla v_1, \dots, u \cdot \nabla v_d)$, $\operatorname{div} u := \sum_{i=1}^{d} \partial_{x_i} u_i$ and h is a fixed function. Further, $\mathcal{G}$ is an appropriate linear operator imposing the boundary conditions.

In the case Ω is a compact Riemannian manifold, either with or without boundary, the Navier–Stokes equation reads

$$\partial_t u + \nabla^1_u u + \nu \Delta_\Omega u + \nabla_\Omega p + h = 0, \quad \operatorname{div} u = 0, \quad \mathcal{G}u|_\Gamma = 0, \quad u(0, x) = u_0(x). \qquad (2)$$

That is we just replace the Laplace operator by the Laplace–de Rham operator, the gradient operator by the Riemannian gradient operator, and the nonlinear term by the Levy-Civita connection. Recall that a flat (Euclidean) domain $\Omega \subset \mathbb{R}^d$ can be seen a Riemannian manifold and we have $-\Delta = \Delta_\Omega$, $\nabla = \nabla_\Omega$ and $\langle u \cdot \nabla \rangle v = \nabla^1_u v$ (see, e.g., [41, Chapter 5]). That is, (2) reads (1) in the Euclidean case. We should say that some authors consider the Navier–Stokes equation on a Riemannian manifold with a slightly different Laplacian operator and sometimes with on more term involving the (Ricci) curvature of the Riemannian manifold. In that case, we also recover (1) in the Euclidean case because the curvature vanishes. Writing the Navier–Stokes as (2), we are following [7,14,20,21,40,41]; for other writings we refer to [10,37].

Often system (2) can be rewritten as an evolutionary system

$$\dot{u} + B(u, u) + Au + h = 0, \quad u(0, x) = u_0(x); \qquad (3)$$

where formally $B(u, v) := \Pi \nabla^1_u v$ and $Au = \nu \Pi \Delta_\Omega u$, and Π is a projection onto a suitable subspace H of divergence free vector fields (cf. [16, Chapter II, Section 3], [39, Section 4], [41, Section 5.5]). Usually $\Pi \nabla = 0$, and we suppose that $h = \Pi h$ (otherwise we have just to take Πh in (3) instead).

The aim of this work is to give some sufficient conditions to guarantee that the solution of system (2) lives in a Gevrey regularity space.

For the case of periodic boundary conditions, that is, for the case $\Omega = \mathbb{T}^d$, the Gevrey regularity has been proven in the pioneering work [17] for the Gevrey class $\mathrm{D}(A^{\frac{1}{2}} \mathrm{e}^{\varphi(t) A^{\frac{1}{2}}})$, provided $u_0 \in \mathrm{D}(A^{\frac{1}{2}})$. Here $\varphi(t) = \min\{\sigma, t\}$, with $\sigma > 0$ fixed depending on the external forcing h. These results have been extended to other Gevrey classes in [35], namely $\mathrm{D}(A^s \mathrm{e}^{\varphi(t) A^{\frac{1}{2}}})$, provided $u_0 \in \mathrm{D}(A^s)$, with $s > \frac{d}{4}$. The first observation is that there is a gap, for the value of s, for $d = 3$. This gap is filled in [5, Theorem 4.1]. Though we consider here bounded domains, we would like to mention the work [6] where unbounded domains $\Omega = \mathbb{R}^l \times \mathbb{T}^m$, $(l, m) \in \mathbb{N}^2 \setminus \{(0, 0)\}$ are considered. Further we refer to [28], where the Gevrey regularity is used to study the level sets of the (scalar) vorticity, in the case $\Omega = \mathbb{T}^2$.

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