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Andô dilations and inequalities on noncommutative varieties ☆

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ABSTRACT

Andô proved a dilation result that implies his celebrated inequality which says that if T_1 and T_2 are commuting contractions on a Hilbert space, then for any polynomial p in two variables,

$$\|p(T_1, T_2)\| \le \|p\|_{\mathbb{D}^2},$$

where $\mathbb{D}^2$ is the bidisk in $\mathbb{C}^2$. The main goal of the present paper is to find analogues of Andô's results for the elements of the bi-ball $\mathbf{P}_{n_1,n_2}$ which consists of all pairs $(\mathbf{X}, \mathbf{Y})$ of row contractions $\mathbf{X} := (X_1, \dots, X_{n_1})$ and $\mathbf{Y} := (Y_1, \dots, Y_{n_2})$ which commute, i.e. each entry of $\mathbf{X}$ commutes with each entry of $\mathbf{Y}$. The results are obtained in a more general setting, namely, when $\mathbf{X}$ and $\mathbf{Y}$ belong to noncommutative varieties $\mathcal{V}_1$ and $\mathcal{V}_2$ determined by row contractions subject to constraints such as

$$q(X_1, \dots, X_{n_1}) = 0 \quad \text{and} \quad r(Y_1, \dots, Y_{n_2}) = 0, \quad q \in \mathcal{P}, r \in \mathcal{R},$$

respectively, where $\mathcal{P}$ and $\mathcal{R}$ are sets of noncommutative polynomials. We obtain dilation results which simultaneously generalize Sz.-Nagy dilation theorem for contractions, Andô's dilation theorem for commuting contractions, Sz.-Nagy–Foiaş commutant lifting theorem, and Schur's representation for the unit ball of H^∞, in the framework of noncommutative

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varieties and Poisson kernels on Fock spaces. This leads to one of the main results of the paper, an Andô type inequality on noncommutative varieties, which, in the particular case when $n_1 = n_2 = 1$ and T_1 and T_2 are commuting contractive matrices with spectrum in the open unit disk $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$, takes the form

$$\begin{aligned}&\|p(T_1, T_2)\| \\ &\quad \le \min\left\{\|p(B_1 \otimes I_{\mathbb{C}^{d_1}}, \varphi_1(B_1))\|, \|p(\varphi_2(B_2), B_2 \otimes I_{\mathbb{C}^{d_2}})\|\right\},\end{aligned}$$

where $(B_1 \otimes I_{\mathbb{C}^{d_1}}, \varphi_1(B_1))$ and $(\varphi_2(B_2), B_2 \otimes I_{\mathbb{C}^{d_2}})$ are analytic dilations of (T_1, T_2) while B_1 and B_2 are the universal models associated with T_1 and T_2, respectively. In this setting, the inequality is sharper than Andô's inequality and Agler–McCarthy's inequality. We obtain more general inequalities for arbitrary commuting contractive matrices and improve Andô's inequality for commuting contractions when at least one of them is of class $\mathcal{C}_0$.

We prove that there is a universal model $(S \otimes I_{\ell^2}, \varphi(S))$, where S is the unilateral shift and $\varphi(S)$ is an isometric analytic Toeplitz operator on $H^2(\mathbb{D}) \otimes \ell^2$, such that

$$\|[p_{rs}(T_1, T_2)]_k\| \le \|[p_{rs}(S \otimes I_{\ell^2}, \varphi(S))]_k\|,$$

for any commuting contractions T_1 and T_2 on Hilbert spaces, any $k \times k$ matrix $[p_{rs}]_k$ of polynomials in $\mathbb{C}[z, w]$, and any $k \in \mathbb{N}$. Analogues of this result for the bi-ball $\mathbf{P}_{n_1,n_2}$ and for a class of noncommutative varieties are also considered.

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0. Introduction

Two of the most important results in operator theory are von Neumann's inequality [28] and Andô's generalization [2] to two commuting contractions on a Hilbert space (see also [26,5,6], and [7]). Andô's inequality is essentially equivalent (see [11] and [12]) to the Sz.-Nagy–Foiaş commutant lifting theorem [25] and states that if T_1 and T_2 are commuting contractions on a Hilbert space, then for any polynomial p in two variables,

$$\|p(T_1, T_2)\| \le \|p\|_{\mathbb{D}^2},$$

where $\mathbb{D}^2$ is the bidisk in $\mathbb{C}^2$. Varopoulus [27] found a counterexample showing that the inequality does not extend to three mutually commuting contractions. For a nice survey and further generalizations of these inequalities we refer to Pisier's book [13].

In a remarkable paper [1], Agler and McCarthy improved Andô's inequality in the case of contractive matrices with no eigenvalues of modulus 1. They showed that

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