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Blow-up criterion for the incompressible viscoelastic flows

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In this paper, we are concerned with the incompressible viscoelastic flows in the periodic domain. We establish a Serrin-type blow-up criterion for 3-D periodic initial boundary problem, which states a strong solution exists globally, provided that the velocity satisfies Serrin's condition and the $L_t^\infty L_x^\infty$-norm of the deformation gradient are bounded. We also establish blow-up criterion in terms of the upper bound of the deformation gradient for 2-D periodic initial boundary problem. The main ingredient of the proof is a priori estimate for an important quantity under the assumption that the deformation gradient is upper bounded, whose divergence can be viewed as the effective viscous flux.

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1. Introduction

The flow of incompressible viscoelastic fluids can be described by the following (cf. [5,6,16,18,19]):

$$\begin{cases} \partial_t u + u \cdot \nabla F - \mu \Delta u + \nabla P = \operatorname{div}(FF^\top), \\ \partial_t F + u \cdot \nabla F = \nabla u F, \\ \operatorname{div} u = 0. \end{cases} \tag{1.1}$$

Here $u \in \mathbb{R}^d$ $(d = 2, 3)$ denotes the velocity of the fluid, $F \in \mathcal{M}$ is the deformation gradient ($\mathcal{M}$ is the set of $d \times d$ matrices with $\det F = 1$), and P is the pressure of the fluid, which is a Lagrangian multiplier due to the incompressibility of the fluid $\operatorname{div} u = 0$. The viscosity μ is a positive constant, and without loss of generality, will be assumed to be 1 here.

In this paper, we consider the system (1.1) supplemented with the following conditions,

$$(u, F)|_{t=0} = (u_0, F_0)(x), \quad x \in \Omega, \tag{1.2}$$

$$⨍_\Omega u(t, x)dx = 0, \quad t > 0, \tag{1.3}$$

where $\Omega = \mathbb{T}^d$ with $d = 2$ or 3 denotes the physical domain under consideration.

Let's first review some of the previous works in this direction. The issues on the well-posedness of the system (1.1) have been extensively studied recently. For the local well-posedness with general initial data, we refer the readers to Lin et al. [18], Chen et al. [3], Lei et al. [16], and Lin et al. [19]. The results mentioned above were proved by the standard energy method in Hilbert space H^s. It is worth noting that the method employed by Lei et al. [16] is independent of the space dimensions. Indeed, they studied both the Cauchy problem in $\mathbb{R}^n$ and the periodic problem in the n-dimensional torus $\mathbb{T}^n$ of the system (1.1) with the initial data u_0 and F_0 satisfying the following constraints:

$$\begin{cases} \nabla \cdot u_0 = 0, \\ \det(F_0) = 1, \\ \nabla \cdot F_0 = 0, \\ \nabla_m F_{0ij} - \nabla_j F_{0im} = F_{0lj} \nabla_l F_{0im} - F_{0lm} \nabla_l F_{0ij}. \end{cases} \tag{1.4}$$

The authors in [3,16,18,19] also studied the global well-posedness of the system in Hilbert space H^s. However, the results allow only small initial data, in the sense that the initial data is a H^2 small perturbation around the equilibrium $(0, I)$, where I is the identity matrix. For related discussions on solution with small initial data, see also [6,10,12, 14,15,22–24] and references therein. For the corotational Oldroyd-B model with a finite relaxation time, the global existence of weak solutions with arbitrary initial data had been