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On stabilizing effect of elasticity in the Rayleigh–Taylor problem of stratified viscoelastic fluids

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ABSTRACT

We investigate the stabilizing effect of elasticity in the Rayleigh–Taylor (RT) problem of stratified immiscible viscoelastic fluids, separated by a free interface and in the presence of a uniform gravitational field, in a horizontally periodic domain where the velocities of the fluids are non-slip on both upper and lower fixed flat boundaries, while the internal surface tension is omitted. We establish a discriminant $\mathrm{C_r}$ for the stability of the stratified viscoelastic RT problem. More precisely, if $\mathrm{C_r} < 1$, then the stratified viscoelastic RT equilibrium state is exponentially stable. This means that a sufficiently large elasticity coefficient has stabilizing effect so that it can inhibit viscoelastic RT instability. On the other hand, if $\mathrm{C_r} > 1$, then we show that the RT equilibrium state is linearly unstable in the Hadamard sense. Moreover, for the case of a nonhomogeneous incompressible viscoelastic fluid, the condition $\mathrm{C_r} > 1$ will lead to the nonlinear instability of the RT equilibrium state; and this shows that the RT

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instability still occurs in viscoelastic fluids when the elasticity coefficient is small.

1. Introduction

The models of viscoelastic fluids formulated by Oldroyd [43,44] (see [4,45,47] for alternative derivations and perspectives), in particular, the Oldroyd-B model, have been studied by many authors (see [19,31,32,39,51] and the references cited therein) since the early work of Renardy [49] and Guillopé and Saut [12]. Here we consider the following version of the three-dimensional (3D) incompressible idea Oldroyd-B model

$$\begin{cases} \rho\partial_t v + \rho v\cdot\nabla v + \nabla p = \mu\Delta v + \kappa \mathrm{div}(\rho U U^{\mathrm{T}}) \\ \partial_t U + v\cdot\nabla U - \nabla v U = 0, \\ \mathrm{div} v = 0, \end{cases} \tag{1.1}$$

where $v(x,t)\in\mathbb{R}^3$ is the fluid velocity, $U(x,t)$ denotes the deformation tensor (a 3×3 matrix-valued function), $p(x,t)$ represents the hydrodynamic pressure, and ρ, μ and κ denote the density, the viscosity coefficient and the elasticity coefficient of the fluid, respectively.

The existence of small solutions to the Cauchy problem and initial boundary value problems for (1.1) has been established by Lin, Liu and Zhang in [36–38], and the corresponding periodic boundary problem has been investigated by Lei et al. in [34,35]. We mention that the corresponding compressible fluid case has been also studied by many authors, see, for example, [20–22,46] and the references cited therein.

In this article we investigate the stability of the stratified (or two-layers) viscoelastic Rayleigh–Taylor (VRT) equilibrium state for the system (1.1) of stratified immiscible viscoelastic fluids, separated by a free interface, in the presence of a uniform gravitational field. We shall consider the stratified VRT problem in a horizontal periodic domain $\Omega\subset\mathbb{R}^3$, in which the velocities of the fluids are non-slip on both upper and lower fixed flat boundaries, and the density of the upper viscoelastic fluid is heavier than the density of the lower one, while the internal surface tension will be omitted. Next, we formulate the VRT problem in details.

1.1. Stratified VRT problem in Eulerian coordinates

Referring to the derivation of (1.1) and the motion equations of stratified viscous fluids (see [27] for example), we can write the stratified VRT problem without internal surface tension for (1.1) as follows.