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Kernels of Toeplitz operators on the Hardy space over the bidisk ☆

Yong Chen [a], Kei Ji Izuchi [b], Young Joo Lee [c,*]

[a] Department of Mathematics, Zhejiang Normal University, Jinhua, 321004, PR China
[b] Department of Mathematics, Niigata University, Niigata, 950-2181, Japan
[c] Department of Mathematics, Chonnam National University, Gwangju, 61186, Republic of Korea

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ABSTRACT

We study the kernels of Toeplitz operators on the Hardy space on the bidisk. We first give a sufficient condition for a general symbol to be antiholomorphic under the assumption that the kernel of the corresponding Toeplitz operator contains a backward shift invariant subspace. As an application, we construct an invariant subspace whose its orthogonal complement can not be the kernel of any Toeplitz operator. Also, we give a characterization on a Toeplitz operator for which the orthogonal complement of its kernel is generated by certain inner functions. Finally, we describe all backward shift invariant subspaces which are in the kernels of Toeplitz operators with homogeneous type symbols. As an application, we show that there is a Toeplitz operator for which its kernel has dimension of any given integer and the orthogonal

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* Corresponding author.
E-mail addresses: ychen@zjnu.edu.cn, ychen227@gmail.com (Y. Chen), izuchi@m.sc.niigata-u.ac.jp (K.J. Izuchi), leeyj@chonnam.ac.kr (Y.J. Lee).

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complement of the kernel is generated by two functions. Our result shows that there are higher dimensional phenomena.
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1. Introduction

Let $\mathbb{T}$ be the boundary of the unit disk $\mathbb{D}$ in the complex plane $\mathbb{C}$. The bidisk $\mathbb{D}^2$ and the torus $\mathbb{T}^2$ are the Cartesian product of 2 copies of $\mathbb{D}$ and $\mathbb{T}$ respectively. We let $L^p = L^p(\mathbb{T}^2, \sigma)$ be the usual Lebesgue space of $\mathbb{T}^2$ where σ is the normalized Haar measure on $\mathbb{T}^2$. The Hardy space $H^2 = H^2(\mathbb{D}^2)$ is the Hilbert space of holomorphic functions f on $\mathbb{D}^2$ satisfying

$$\|f\|^2 := \sup_{0<r<1} \int_{\mathbb{T}^2} |f(r\zeta)|^2 \, d\sigma(\zeta) < \infty.$$

As is well known, for a function f in $H^2(\mathbb{D}^2)$ there is the boundary function $\lim_{r\to 1} f(r\zeta)$ for almost every $\zeta \in \mathbb{T}^2$ which is contained in L^2, and its Poisson extension coincides with f on $\mathbb{D}^2$. So identifying a function in $H^2(\mathbb{D}^2)$ with its boundary function, we think of $H^2(\mathbb{D}^2)$ as $H^2(\mathbb{D}^2) \subset L^2$. Let P denote the orthogonal projection from L^2 onto H^2. For a function $u \in L^\infty$, the *Toeplitz operator* T_u with symbol u is defined by

$$T_u f = P(uf)$$

for functions $f \in H^2$. Then clearly T_u is a bounded linear operator on H^2.

We will usually write (z, w) to denote a point in $\mathbb{D}^2$ or $\mathbb{T}^2$. Also, we denote by z and w the coordinate functions. For a closed subspace M of H^2, M is said to be invariant if $T_z M \subset M$ and $T_w M \subset M$. Then, one can see that $H^2 \ominus M$ is a backward shift invariant subspace of H^2, that is $T_z^*(H^2 \ominus M) \subset H^2 \ominus M$ and $T_w^*(H^2 \ominus M) \subset H^2 \ominus M$. Here, L^* denotes the adjoint operator of a bounded operator L. For a subset E of H^2, we denote by $[E]$ the smallest invariant subspace of H^2 containing E. If $M = [f]$ for some $f \in H^2$, we say M is singly generated. Generally, the structure of invariant subspaces is fairly complicated; see [2,9].

In this paper, we study the kernels of Toeplitz operators on H^2. For the Hardy space on the unit disk, the kernels of Toeplitz operators have been well studied. In order to have some motivation from the known results on the unit disk, we briefly introduce Toeplitz operator acting on the Hardy space of the unit disk. Let $H^2(\mathbb{D})$ denote the Hardy space on $\mathbb{D}$ and $L^p(\mathbb{T}) = L^p(\mathbb{T}, \sigma_1)$ denote the usual Lebesgue space on $\mathbb{T}$ where σ_1 is the normalized Lebesgue measure on $\mathbb{T}$. For a given $u \in L^\infty(\mathbb{T})$, the 1-dimensional Toeplitz operator t_u with symbol u is the bounded linear operator on $H^2(\mathbb{D})$ defined by $t_u f = Q(uf)$ for functions $f \in H^2(\mathbb{D})$ where Q is the orthogonal projection from $L^2(\mathbb{T})$ onto $H^2(\mathbb{D})$. We also use the same notions on $\mathbb{D}$ as $\mathbb{D}^2$. Let z be the coordinate

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