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The classification of some generalised Bunce–Deddens algebras ☆

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ABSTRACT

We use K-theory to prove an isomorphism theorem for a large class of generalised Bunce–Deddens algebras constructed by Kribs and Solel from a directed graph E and a sequence ω of positive integers. In particular, we compute the torsion-free component of the K_0-group for a class of generalised Bunce–Deddens algebras to show that supernatural numbers are a complete invariant for this class.

1. Introduction

In [7] Kribs and Solel introduced a family of direct limit C^*-algebras constructed from directed graphs E and sequences $\omega = (n_k)_{k=1}^{\infty}$ of natural numbers such that $n_k | n_{k+1}$ for all $k \in \mathbb{N}$. They named these C^*-algebras generalised Bunce–Deddens algebras.

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The graph E consisting of a single vertex connected by a single loop edge generates the classical Bunce–Deddens algebras.

Supernatural numbers have been used to classify UHF algebras ([4, Theorem 1.12]) and the classical Bunce–Deddens algebras ([1, Theorem 3.7] and [2, Theorem 4]). Kribs showed in [6, Theorem 5.1] that the generalised Bunce–Deddens algebras corresponding to the graph B_N consisting of a single vertex with N loops, are classified by their associated supernatural numbers in the sense that $C^*(B_N, \omega) \cong C^*(B_N, \omega')$ if and only if $[\omega] = [\omega']$. The special case $N = 1$ is Bunce and Deddens' theorem. Kribs and Solel later showed in [7, Theorem 7.5] that the generalised Bunce–Deddens algebras corresponding to the simple cycle with j edges, are classified by their associated supernatural numbers; again the special case $j = 1$ is the original result of Bunce and Deddens. Kribs and Solel asked in [7, Remark 7.7] for what class of graphs E a similar classification theorem could be obtained. Here we prove that such a theorem can be obtained for the class of generalised Bunce–Deddens algebras corresponding to a given strongly connected finite directed graph E such that 1 is an eigenvalue of the vertex matrix, and the only roots of unity that are eigenvalues are the $\mathcal{P}_E$-th roots of unity, where $\mathcal{P}_E$ is the period of the graph E.

In [10, Proposition 3.11] it was shown that if $[\omega] = [\omega']$ then $C^*(E, \omega) \cong C^*(E, \omega')$ for row-finite directed graphs E with no sinks or sources. The main result of this article (Theorem 6.1) shows that if $C^*(E, \omega) \cong C^*(E, \omega')$ then $[\omega] = [\omega']$ for strongly connected finite directed graphs E such that 1 is an eigenvalue of A_E^t and such that the only roots of unity that are eigenvalues of A_E^t are the $\mathcal{P}_E$-th roots of unity. We prove this by studying the torsion-free component of $K_0(C^*(E, \omega))$; we assume that 1 is an eigenvalue of A_E^t to ensure that this is nontrivial. The Perron–Frobenius theorem (see [3, Theorem 8.2.1]) says that if 1 is an eigenvalue of A_E^t, then the $\mathcal{P}_E$-th roots of unity are also eigenvalues of A_E^t. The hypothesis that these are the only roots of unity that are eigenvalues of A_E^t is nontrivial. The *nonnegative inverse eigenvalue problem* asks which sets of n complex numbers $\lambda_1, \ldots, \lambda_n$ occur as the eigenvalues of some $n \times n$ nonnegative matrix. Deep results of [5] regarding this problem show that it is possible for any collection of roots of unity to appear as eigenvalues of a nonnegative matrix.

If 1 is not an eigenvalue of A_E^t, then $K_0(C^*(E, \omega))$ is purely torsion and another argument (perhaps along the lines of [6, Theorem 5.1]) will be needed. We have not addressed that case in this article.

We begin in Section 3 with some calculations for the sums of powers of matrices and about cokernels. We show that the matrix $\sum_{i=0}^{n_k/l-1} (A_E^{il})^t$, where $l := \lim_{j\to\infty} \gcd(\mathcal{P}_E, n_j)$ and $\gcd(\mathcal{P}_E, n_k) = l$, is invertible if the only eigenvalues of A_E^t are the $\mathcal{P}_E$-th roots of unity (Lemma 3.2). We recall the equivalence relation $\sim_l$ on E^0 established in [10, Lemma 4.2] to show that $\operatorname{coker}(1 - A_E^l)^t \cong \bigoplus_{i=1}^{l} \operatorname{coker}(1 - A_E^t)$ (Corollary 3.6).

In Section 4 we compute $K_1(C^*(E, \omega))$ for strongly connected finite directed graphs E such that the only roots of unity that are eigenvalues of A_E^t are the $\mathcal{P}_E$-th roots of unity. We show that the torsion-free component is isomorphic to l copies of $K_1(C^*(E))$ (Theorem 4.1). We do this by showing that $\ker(1 - A_E(n))^t \cong \ker(1 - A_E^n)^t$ for $n \geq 1$