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Bloch functions on bounded symmetric domains

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ABSTRACT

We introduce and characterize Bloch functions on bounded symmetric domains, which may be infinite dimensional, by extending several well-known equivalent conditions for Bloch functions on the open unit disc U in $\mathbb{C}$. We also generalize a number of results concerning Bloch functions on U to bounded symmetric domains. Given a holomorphic mapping φ between bounded symmetric domains $\mathbb{B}_X$ and $\mathbb{B}_Y$, we derive criteria for boundedness and compactness of the composition operator C_φ between the Bloch spaces $\mathcal{B}(\mathbb{B}_Y)$ and $\mathcal{B}(\mathbb{B}_X)$, extending several known results for finite dimensional domains.

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1. Introduction

The classical Bloch functions on the open unit disc in $\mathbb{C}$ play an important role in geometric function theory and have been widely studied. The concept of a Bloch function has been extended to various complex domains in finite or infinite dimensions. In particular, it has been extended by Hahn [10] and Timoney [25,26] to bounded homogeneous domains in $\mathbb{C}^n$, and to infinite dimensional Hilbert balls by Blasco, Galindo and Miralles [3], where a Hilbert ball is the open unit ball of a Hilbert space and is a rank one bounded symmetric domain. One of the interesting topics concerning Bloch functions is that of composition operators. It has been shown by Shi and Luo [24] that composition operators on the Bloch space of bounded homogeneous domains in $\mathbb{C}^n$ are bounded and they gave a sufficient condition for these operators to be compact. They showed further that the latter condition was also necessary for the Euclidean balls. This condition was later proved necessary for the classical Cartan domains by Zhou and Shi [30]. Recently, Dai [8] obtained further equivalent conditions for the compactness of composition operators on the Bloch space of the Euclidean unit ball.

Following our recent study in [6] and [11] of $\mathbb{C}^n$*-valued* Bloch mappings on finite dimensional bounded symmetric domains, our objective in this paper is to focus on *complex-valued* Bloch functions on bounded symmetric domains which, however, can be infinite dimensional. We introduce the concept of a Bloch function on a possibly infinite dimensional bounded symmetric domain and show in Theorem 3.8 that many equivalent conditions for Bloch functions on the unit disc $U = \{\zeta \in \mathbb{C} : |\zeta| < 1\}$ are also equivalent on bounded symmetric domains of all dimensions, including the cases studied in [3,25]. This enables us to extend a number of results concerning Bloch functions on the unit disc U to bounded symmetric domains in Section 3 and Section 5. In particular, we show in Theorem 5.1 and Proposition 5.2 boundedness of composition operators between Bloch spaces and prove various criteria, in Propositions 5.3 and 5.4, Corollary 5.5, Proposition 5.6, Theorems 5.8 and 5.9, Propositions 5.10 and 5.11, for compactness of these operators, extending the aforementioned results in [8,24,30] for finite dimensional domains.

An essential difference from the finite dimensional case is the substitute for the Bergman metric with the Kobayashi metric in infinite dimensional domains and the extensive use of the Bergman operators. To achieve our results, we make use of the underlying Jordan structures of bounded symmetric domains. This is facilitated by Kaup's Riemann mapping theorem [16] which asserts that a bounded symmetric domain is biholomorphic to the open unit ball of a JB*-triple, which is a complex Banach space equipped with a Jordan triple structure. More precisely, a complex Banach space X is called a *JB*-triple* if it admits a continuous *Jordan triple product* $\{\cdot,\cdot,\cdot\} : X^3 \longrightarrow X$ which is symmetric and linear in the outer variables, but conjugate linear in the middle variable, and satisfies

(i) $\{x, y, \{a, b, c\}\} = \{\{x, y, a\}, b, c\} - \{a, \{y, x, b\}, c\} + \{a, b, \{x, y, c\}\}$;