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Structure theory of singular spaces ☆

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A B S T R A C T

In this paper we develop a structure theory of Einstein manifolds or manifolds with lower Ricci curvature bounds for certain singular spaces that arise as geometric limits of sequences of Riemannian manifolds. This theory generalizes the results that were obtained by Cheeger, Colding and Naber in the smooth setting. In the course of the paper, we will carefully characterize the assumptions that we have to impose on this sequence of Riemannian manifolds in order to guarantee that the individual results hold.

An important aspect of our approach is that we don't need impose any Ricci curvature bounds on the sequence of Riemannian manifolds leading to the singular limit. The Ricci curvature bounds will only be required to hold on the regular part of the limit and we will not impose any (synthetic) curvature condition on its singular part.

The theory developed in this paper will have applications in the blowup analysis of certain geometric equations in which we study scales that are much larger than the local curvature scale. In particular, this theory will have applications in the study of Ricci flows of bounded scalar curvature, which we will describe in a subsequent paper.

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Contents

1. Introduction and statement of the main results

1.1. Introduction

Geometric limit and, in particular, blowup arguments have become popular tools in the study of geometric PDEs. Useful applications of these tools can be found in the analysis of intrinsic objects, such as Einstein metrics and Ricci flows, or extrinsic objects, such as minimal surfaces and mean curvature flow. The goal behind limit and blowup arguments is often to gain approximate characterizations of solutions of the geometric PDEs under investigation, at small scales. In a broad sense, the strategy of proof is the following: One first shows compactness of blowups of these solutions in an appropriate topology, possibly under additional assumptions. Consequently, any sequence of solutions subconverges to a limit space, which often exhibits additional geometric properties. If arguments are set up adequately, then these additional properties may be used to analyze these limit spaces more deeply, which may give rise to extra structural information or geometric bounds. This extra information can then be used to derive geometric bounds or local characterizations of the actual solutions that led to the limit space.

Often, however, the conditions under which a geometric limit can be extracted are rather restrictive. For example, in many situations it is necessary to assume that the solutions of the geometric PDE under investigation satisfy uniform curvature bounds, in order to ensure that the limit space is smooth. These restrictions usually confine our

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