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Kantorovich duality for general transport costs and applications [☆]



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ABSTRACT

We introduce a general notion of transport cost that encompasses many costs used in the literature (including the classical one and weak transport costs introduced by Talagrand and Marton in the 90's), and prove a Kantorovich type duality theorem. As a by-product we obtain various applications in different directions: we give a short proof of a result by Strassen on the existence of a martingale with given marginals, we characterize the associated transport-entropy inequalities together with the log-Sobolev inequality restricted to convex/concave functions. We also provide explicit examples of discrete measures satisfying the weak transport-entropy inequalities derived here.

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1. Introduction

Concentration of measure phenomenon was introduced in the seventies by V. Milman [\[46\]](#) in his study of asymptotic geometry of Banach spaces. It was then studied in depth by many authors including Gromov [\[32,31\]](#), Talagrand [\[63\]](#), Maurey [\[44\]](#), Ledoux [\[36,10\]](#), Bobkov [\[6,11\]](#) and many others and played a decisive role in analysis, probability and statistics in high dimensions. We refer to the monographs [\[37\]](#) and [\[16\]](#) for an overview of the field.

One classical example of such a phenomenon can be observed for the standard Gaussian measure γ_m on \mathbb{R}^m . It follows from the well-known Sudakov–Tsirelson–Borell isoperimetric result in Gauss space [\[62,15\]](#) that if X_1, \dots, X_n are n i.i.d random vectors with law γ_m and $f : (\mathbb{R}^m)^n \rightarrow \mathbb{R}$ is a 1-Lipschitz function (with respect to the Euclidean norm), then

$$\mathbb{P}(f(X_1, \dots, X_n) > \text{med}(f) + t) \leq e^{-(t-t_0)^2/(2a)}, \quad \forall t \geq t_0, \quad (1.1)$$

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