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A new approach to low-distortion embeddings of finite metric spaces into non-superreflexive Banach spaces

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A B S T R A C T

The main goal of this paper is to develop a new embedding method which we use to show that some finite metric spaces admit low-distortion embeddings into all non-superreflexive spaces. This method is based on the theory of equal-signs-additive sequences developed by Brunel and Sucheston (1975–1976). We also show that some of the low-distortion embeddability results obtained using this method cannot be obtained using the method based on the factorization between the summing basis and the unit vector basis of ℓ_1, which was used by Bourgain (1986) and Johnson and Schechtman (2009).

1. Introduction

One of the basic problems of the theory of metric embeddings is: given some Banach space or a natural class $\mathcal{P}$ of Banach spaces find classes of metric spaces which admit

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low-distortion embeddings into each Banach space of the class $\mathcal{P}$. The main goal of this paper is to develop a new embedding method which can be used to show that some finite metric spaces admit low-distortion embeddings into all non-superreflexive spaces (Theorem 1.3). This method is based on the theory of equal-signs-additive sequences (ESA) developed by Brunel and Sucheston [8–10]. We show in Theorem 1.6 that some of the low-distortion embeddability results obtained using this method cannot be obtained using the method based on the factorization between the summing basis and the unit vector basis of ℓ_1, which was used by Bourgain [6] and Johnson and Schechtman [20], see Corollary 1.5.

The problem mentioned at the beginning of the previous paragraph can be regarded as one side of the problem of metric characterization of the class $\mathcal{P}$. Recall that, in the most general sense, a *metric characterization* of a class of Banach spaces is a characterization which refers only to the metric structure of a Banach space and does not involve the linear structure. The study of metric characterizations became an active research direction in mid-1980s, in the work of Bourgain [6] and Bourgain, Milman, and Wolfson [7] (see also Pisier [45, Chapter 7]). The work on metric characterization of isomorphic invariants of Banach spaces determined by their finite-dimensional subspaces, and on generalization of the obtained theory to general metric spaces became known as the *Ribe program*, see [2,36]. The type of metric characterizations which is closely related to the present paper is the following:

Definition 1.1 *([40])*. Let $\mathcal{P}$ be a class of Banach spaces and let $T = \{T_\alpha\}_{\alpha\in A}$ be a set of metric spaces. We say that T is a set of *test-spaces* for $\mathcal{P}$ if the following two conditions are equivalent for a Banach space X:

(1) $X \notin \mathcal{P}$;
(2) The spaces $\{T_\alpha\}_{\alpha\in A}$ admit bilipschitz embeddings into X with uniformly bounded distortions.

There are several known different sets of finite test-spaces for superreflexivity of Banach spaces, including: the set of all finite binary trees (Bourgain [6], see also [32,21]), the set of diamond graphs, and the set of Laakso graphs (Johnson and Schechtman [20], see also [38]). In [41,37,29] it was shown that these sets of test-spaces are independent in the sense that the respective families of metric spaces do not admit bilipschitz embeddings one into another with uniformly bounded distortions.

There are also metric characterizations of superreflexivity using only one metric test-space. Baudier [3] proved that the infinite binary tree is such a test-space, many other one-element test-spaces for superreflexivity were described in [41]. See [42] for a survey on metric characterizations of superreflexivity.

The first main result of the present paper is a construction of bilipschitz embeddings with a uniform bound on distortions of diamond graphs with arbitrary finite number of branches into any non-superreflexive Banach space. Multibranching diamonds are a

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