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Non-spectral problem for the planar self-affine measures ☆

Jing-Cheng Liu [a], Xin-Han Dong [a,*], Jian-Lin Li [b]

[a] *Key Laboratory of High Performance Computing and Stochastic Information Processing (Ministry of Education of China), College of Mathematics and Computer Science, Hunan Normal University, Changsha, Hunan 410081, China*

[b] *School of Mathematics and Information Science, Shaanxi Normal University, Xi'an 710119, China*

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In this paper, we consider the non-spectral problem for the planar self-affine measures $\mu_{M,D}$ generated by an expanding integer matrix $M \in M_2(\mathbb{Z})$ and a finite digit set $D \subset \mathbb{Z}^2$. Let $p \geq 2$ be a positive integer, $E_p^2 := \frac{1}{p}\{(i,j)^t : 0 \leq i,j \leq p-1\}$ and $\mathcal{Z}_D^2 := \{x \in [0,1)^2 : \sum_{d\in D} e^{2\pi i\langle d,x\rangle} = 0\}$. We show that if $\emptyset \neq \mathcal{Z}_D^2 \subset E_p^2 \setminus \{0\}$ and $\gcd(\det(M), p) = 1$, then there exist at most p^2 mutually orthogonal exponential functions in $L^2(\mu_{M,D})$. In particular, if p is a prime, then the number p^2 is the best.

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* Corresponding author.
E-mail addresses: jcliu@hunnu.edu.cn (J.-C. Liu), xhdong@hunnu.edu.cn (X.-H. Dong), jllimath10@snnu.edu.cn (J.-L. Li).

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1. Introduction

Let $M \in M_n(\mathbb{R})$ be an $n \times n$ expanding real matrix (that is, all the eigenvalues of M have moduli > 1), and $D \subset \mathbb{R}^n$ be a finite subset with cardinality $\#(D)$. Let $\{\phi_d(x)\}_{d\in D}$ be an iterated function system (IFS) defined by

$$\phi_d(x) = M^{-1}(x + d) \ \ (x \in \mathbb{R}^n, \quad d \in D).$$

Then the IFS arises a natural *self-affine measure* μ satisfying

$$\mu = \mu_{M,D} = \frac{1}{\#(D)} \sum_{d\in D} \mu \circ \phi_d^{-1}. \tag{1.1}$$

Such a measure $\mu_{M,D}$ is supported on the attractor $T(M, D)$ of the IFS $\{\phi_d\}_{d\in D}$ [8].

For a countable subset $\Lambda \subset \mathbb{R}^n$, let $\mathcal{E}_\Lambda = \{e^{2\pi i\langle\lambda,x\rangle} : \lambda \in \Lambda\}$. We call μ a spectral measure, and Λ a spectrum of μ if $\mathcal{E}_\Lambda$ is an orthogonal basis for $L^2(\mu)$. We also say that (μ, Λ) is a *spectral pair*. The existence of a spectrum for μ is a basic problem in harmonic analysis, it was initiated by Fuglede in his seminal paper [7]. The first example of a singular, non-atomic, spectral measure was given by Jorgensen and Pedersen in [10]. This surprising discovery has received a lot of attention, and the research on the spectrality of self-affine measures has become an interesting topic. Also, new spectral measures were found in [9,1–6,13,14] and references cited therein. A related problem is the non-spectral problem of self-affine measure. In [4], Dutkay and Jorgensen showed that if $M = \begin{bmatrix} p & 0 \\ 0 & p \end{bmatrix}$ with $p \in \mathbb{Z} \setminus 3\mathbb{Z}$, $p \geq 2$ and

$$\mathscr{D} = \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}, \tag{1.2}$$

then there are no 4 mutually orthogonal exponential functions in $L^2(\mu_{M,\mathscr{D}})$; they also proved that if $M = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$, then there exist at most 7 mutually orthogonal exponential functions in $L^2(\mu_{M,\mathscr{D}})$. In [11], the third author of this paper proved that if the expanding integer matrix $M = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$ with $ac \notin 3\mathbb{Z}$, then there exist at most 3 mutually orthogonal exponential functions in $L^2(\mu_{M,\mathscr{D}})$, and the number 3 is the best. The third author also obtained the same conclusions for the expanding integer matrix $M = \begin{bmatrix} a & b \\ d & c \end{bmatrix}$ with $\det(M) = ac - bd \notin 3\mathbb{Z}$ in [12]. In this paper, we will give a more general result which is suitable for more self-affine measures. Before the statement of the main results, we first give some definitions and notations.

For a positive integer $p \geq 2$ and a finite digit set $D \subset \mathbb{Z}^n$, let