Journal of Functional Analysis 272 (2017) 3965–3986

Contents lists available at ScienceDirect

Journal of Functional Analysis

www.elsevier.com/locate/jfa

Existence of Lipschitz continuous solutions to the Cauchy–Dirichlet problem for anisotropic parabolic equations

Alkis S. Tersenov [a,c,*], Aris S. Tersenov [b,c]

[a] University of Crete, Department of Mathematics, 71003 Heraklion, Crete, Greece
[b] Sobolev Institute of Mathematics, 630090 Novosibirsk, Russia
[c] Novosibirsk State University, 630090 Novosibirsk, Russia

A R T I C L E I N F O

Article history:
Received 30 October 2014
Accepted 18 February 2017
Available online 2 March 2017
Communicated by Cédric Villani

MSC:
35K20
35K65
35B45

Keywords:
Singular parabolic equations
Degenerate parabolic equations

A B S T R A C T

The Cauchy–Dirichlet and the Cauchy problem for the degenerate and singular quasilinear anisotropic parabolic equations are considered. We show that the time derivative u_t of a solution u belongs to L_∞ under a suitable assumption on the smoothness of the initial data. Moreover, if the domain satisfies some additional geometric restrictions, then the spatial derivatives u_{x_i} belong to L_∞ as well. In the singular case we show that the second derivatives $u_{x_i x_j}$ of a solution of the Cauchy problem belong to L_2.

1. Introduction and main results

Let Ω be a bounded domain in $\mathbf{R}^n$ satisfying the exterior sphere condition and $\Omega_T = (0,T) \times \Omega$ with an arbitrary $T \in (0,\infty)$. We denote by $x = (x_1, \dots, x_n)$ the points in

* Corresponding author.
E-mail addresses: tersenov@math.uoc.gr (A.S. Tersenov), atersenov@math.nsc.ru (A.S. Tersenov).

http://dx.doi.org/10.1016/j.jfa.2017.02.014

Ω and by t the time variable that varies in the interval $[0,T]$. Consider the following quasilinear parabolic equation

$$u_t = \sum_{i=1}^{n} (|u_{x_i}|^{p_i} u_{x_i})_{x_i} \ \text{ in } \ \Omega_T, \tag{1.1}$$

coupled with the homogeneous Dirichlet boundary condition

$$u = 0 \ \text{ on } \ [0,T] \times \partial\Omega \tag{1.2}$$

and the initial condition

$$u(0,x) = u_0(x) \ \text{ in } \ \Omega, \ \ u_0(x) \in C^2(\Omega) \ \text{ and } \ u_0(x) = 0 \ \text{ on } \ \partial\Omega. \tag{1.3}$$

Here $p_i > -1$, $i = 1, \dots, n$. Without loss of generality, we assume that the p_i are ordered:

$$-1 < p_1 \le p_2 \le \ldots \le p_n < +\infty.$$

Let $-1 < p_i < 0$ for $i = 1, \ldots, m$ and $p_i \ge 0$ for $i = m+1, \ldots, n$ where $0 \le m \le n$.

This class of equations has received considerable attention in the last years and not only, see, for example, [1–5,10–13,22,23] and the references therein. Concerning the different aspects of the stationary case, see, for example, [6,7,17,22,23]. From [13] it follows that if $u_0 \in L_\infty(\Omega)$, then there exists a unique weak solution of problem (1.1)–(1.3) which is defined as a function

$$u \in L_\infty(\Omega_T) \cap V(\Omega_T) \cap C([0,T]; L_s(\Omega)) \ \ \forall s \in [1,\infty), \ \ u_t \in V^*(\Omega_T),$$

satisfying the integral identity

$$\int_{\Omega_T} \Big(u\phi_t - \sum_{i=1}^{n} |u_{x_i}|^{p_i} u_{x_i} \phi_{x_i} \Big) dxdt = - \int_{\Omega} u_0 \phi(0,x) dx$$

for an arbitrary smooth function $\phi(t,x)$ which is equal to zero for $x \in \partial\Omega$ and for $t = T$. Here $V^*(\Omega_T)$ is the adjoint space to $V(\Omega_T) = \cap_{i=1}^{n} L_{p_i+2}(0,T; U_i(\Omega))$ where $U_i(\Omega)$ is the closure of $C_\infty^0(\Omega)$ with respect to the norm $\|u\|_{U_i} = \|u\|_{L_2} + \|u_{x_i}\|_{L_{p_i+2}}$ (for more details see [13]).

The main goal of the present paper is to show that under the following assumption on the initial data u_0:

$$\sum_{i=1}^{n} \max_{\Omega} |(|u_{0x_i}|^{p_i} u_{0x_i})_{x_i}| < +\infty, \tag{1.4}$$

the derivative of a solution with respect to t is an L_∞ function. The proof is based on the idea of introducing a new time variable inspired by the idea of introducing a new