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Largest well-posed spaces for the general diffusion system with nonlocal interactions

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ABSTRACT

The authors derive a general diffusion (GD) system with nonlocal interactions of special structure via energetic variational approach and observe that there exist two critical values of s, i.e. $s = \frac{1}{2}, 1$, for the nonlocal interactions, where $s = \frac{1}{2}$ reflects how strong nonlocal property we have and $s = 1$ affects the linearization and choice of initial data spaces. The authors also establish the global existence and uniqueness of mild solution.

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1. Introduction

We study the following N-dimensional ($N \geq 2$) general diffusion system

$$\begin{cases} \rho_t + \nabla \cdot (u\rho) & = 0, \\ \mu_{en} \nabla \rho + \mu_{in}\, \rho \nabla K_{2s} * \rho & = -u\rho, \end{cases} \tag{1.1}$$

where[1] $0 \leq s \leq 1$, $0 < \mu_{en} \leq \infty$, $-\infty < \mu_{in} < \infty$, u is the effective transport velocity vector, and $u\rho$ is the flux that contains nonlocal term $\nabla K_{2s} * \rho$ with $\nabla K_{2s} * \rho = \mathcal{F}^{-1}(i\xi|\xi|^{-2s}\mathcal{F}\rho(\xi))$ in distributional sense, see [11, Chapter 2].

The model arises from the consideration of a continuum density distribution ρ that evolves in time following a velocity field u, according to the continuity equation $\rho_t + \nabla \cdot (u\rho) = 0$ with

$$\int \rho(x,t)dx = \int \rho(x,t)|_{t=0}\, dx$$

for all $t > 0$. Here u is given by the following potential

$$u = -\mu_{en} \nabla \ln \rho - \mu_{in} \nabla K_{2s} * \rho,$$

which arises, for instance, in porous media for $\mu_{in} > 0$ and $s = 0$ according to Darcy's law [6] and chemotaxis for $\mu_{in} < 0$ and $s = 1$ [8,16], respectively.

1.1. Energetic variational approach

In this subsection, we employ the Energetic Variational Approach (EVA) [13] for an isothermal closed system. Hence we can derive from the First Law and Second Law of Thermodynamics the following energy dissipation law:

$$\frac{d}{dt} E^{total} = -\Delta, \tag{1.2}$$

where E^{total} represents the sum of kinetic energy and total Helmholtz free energy, and Δ is the energy dissipation rate/entropy production. As a direct consequence of the choice of total energy functional, dissipation functional, and kinematic relation of the variables employed in the system, one can get all the physics and the assumptions correspondingly.

As a precise framework, one can use the EVA to obtain the force balance equations from the general dissipation law (1.2). Precisely speaking, the Least Action Principle (LAP) determines the Hamiltonian part, and the Maximum Dissipation Principle (MDP) gives the dissipative part. Formally, LAP states that work equals force multiplies distance, i.e.

[1] One of the special cases of $s > 1$, i.e. $s = 2$ and $N = 4$ is also considered.