Journal of Functional Analysis 272 (2017) 4222–4262

Contents lists available at ScienceDirect

Journal of Functional Analysis

www.elsevier.com/locate/jfa

Traveling waves and spreading speeds for time–space periodic monotone systems ☆

Jian Fang [a,b,*], Xiao Yu [c], Xiao-Qiang Zhao [c]

[a] *Department of Mathematics, Harbin Institute of Technology, Harbin 150001, China*
[b] *Ecole des Hautes Etudes en Sciences Sociales, CAMS, 75244 Paris, Cedex 13, France*
[c] *Department of Mathematics and Statistics, Memorial University of Newfoundland, St. John's, NL A1C 5S7, Canada*

A R T I C L E   I N F O

Article history:
Received 23 November 2015
Accepted 28 February 2017
Available online 10 March 2017
Communicated by F.-H. Lin

MSC:
35C07
35K57
37C65
37N25
92D25

Keywords:
Monotone semiflows
Time–space periodicity
Almost pulsating waves
Competitive systems

A B S T R A C T

In this paper, we first establish the existence of traveling waves and spreading speeds for time–space periodic monotone systems with monostable structure via the Poincaré maps approach combined with an evolution viewpoint. Our construction of time–space periodic wave profiles also gives rise to a family of almost pulsating waves, which is a new observation in time and space periodic media. We then apply the developed theory to two species competitive reaction–advection–diffusion systems, and prove that the minimal wave speed exists and coincides with the single spreading speed for such a system no matter whether the spreading speed is linearly determinate. We further obtain a set of sufficient conditions for the linear determinacy.

☆ The research leading to these results has received funding from the European Research Council under the European Union's Seventh Framework Programme (FP/2007–2013)/ERC Grant Agreement No. 321186 – ReaDi – Reaction–Diffusion Equations, Propagation and Modeling. J. Fang's research is supported in part by the National Natural Science Foundation of China and the Fundamental Research Funds for Central Universities. X.-Q. Zhao's research is supported in part by the Natural Science and Engineering Research Council of Canada.

* Corresponding author.
E-mail addresses: jfang@hit.edu.cn (J. Fang), xy3267@mun.ca (X. Yu), zhao@mun.ca (X.-Q. Zhao).

http://dx.doi.org/10.1016/j.jfa.2017.02.028

1. Introduction

Periodic environment of space and/or time is one of the useful approximations to understand the influence of the environmental heterogeneity on the propagation phenomena arising from ecological and biological processes. The study of reaction–diffusion equations in space-periodic media may go back to Freidlin [12] and Freidlin and Gärtner [13]. The notion of pulsating traveling fronts was introduced by Shigesada, Kawasaki and Teramoto [36]. The spatially periodic front solution having the exact form $u(t,x) = U(x, x - ct)$ was first found and constructed by Xin [40]. It is also called a periodically varying wavefront by Hudson and Zinner [19]. These notions of fronts are equivalent when the wave speed is not zero. Berestycki and Hamel [3] established various existence, uniqueness and monotonicity of pulsating fronts for a general reaction–diffusion equation with combustion-type or monostable nonlinearity, see also [5,6]. Hamel and Roques [14] gave a complete classification of all KPP pulsating fronts and obtained some global stability properties for the wave fronts including the one with the minimal speed. Weinberger [38] proved the existence of the minimal wave speed and its coincidence with the spreading speed for a recursion defined by an order-preserving compact operator of monostable type, without assuming a KPP type condition. For a monostable semiflow in one-dimensional periodic environment, Liang and Zhao [25] introduced a topologically conjugate semiflow defined in spatially discrete homogeneous environment and then showed that the spreading speed exists and coincides with the minimal wave speed of the front having the form $U(x, x - ct)$. For monotone semiflows of bistable type, Fang and Zhao [10] interpreted the bistability from a viewpoint of monotone dynamical systems to find a link between the monostable subsystems and the bistable system itself, which is used to establish the existence of bistable wavefronts. We refer to [21,7,34,35] for nonlocal dispersal equations, and two survey papers [41,15] for more references. In time-periodic media, there are also quite a few investigations on traveling waves of reaction–diffusion equations, see, e.g., [1,11,45,43,2,46,26] and references therein. For time-periodic semiflows in one dimensional continuous medium, Liang, Yi and Zhao [23] used the wavefront $W(x - c\omega)$ obtained for the Poincaré map Q_ω to construct a two-variable function $U(t,\xi) := Q_t[W](\xi + ct)$, which is then shown to be a periodic traveling wave for the semiflow $\{Q_t\}_{t\geq 0}$.

In the case where the time and space periodicity is incorporated into a reaction–diffusion equation, it remains unclear whether there exists a transition wave in the sense of Berestycki and Hamel [4], reflecting some interactions of time and space periods. Next we recall some works related to this question. Nolen, Rudd and Xin [30] used a three-variable function $\phi(\xi, t, x)$, which is periodic in the last two arguments, and an auxiliary equation to define a generalized pulsating wave $\phi(x \cdot e - ct, t, x)$, which reduces to a classical pulsating wave in the sense of Xin [40] if the wave speed is *rationally* proportional to the quotient of space and time periods. Moreover, such solutions are also almost planar waves under the setting of generalized transitions waves, for which we refer to [4, Definition 1.5]. Later on, Nadin [28] introduced an equivalent definition of generalized