Accepted Manuscript

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PII: S0022-1236(17)30223-9
DOI: http://dx.doi.org/10.1016/j.jfa.2017.06.006
Reference: YJFAN 7812

To appear in: *Journal of Functional Analysis*

Received date: 20 September 2016
Accepted date: 2 June 2017

Branching Brownian motion with absorption and the all-time minimum of branching Brownian motion with drift

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June 9, 2017

Abstract

We study a dyadic branching Brownian motion on the real line with absorption at 0, drift $\mu \in \mathbb{R}$ and started from a single particle at position $x > 0$. With $K(\infty)$ the (possibly infinite) total number of individuals absorbed at 0 over all time, we consider the functions $\omega_s(x) := \mathbb{E}^x[s^{K(\infty)}]$ for $s \geq 0$. In the regime where μ is large enough so that $K(\infty) < \infty$ almost surely and that the process has a positive probability of survival, we show that $\omega_s < \infty$ if and only if $s \in [0, s_0]$ for some $s_0 > 1$ and we study the properties of these functions. Furthermore, $\omega(x) := \omega_0(x) = \mathbb{P}^x(K(\infty) = 0)$ is the cumulative distribution function of the all time minimum of the branching Brownian motion with drift started at 0 without absorption.

We give descriptions of the family $\omega_s, s \in [0, s_0]$ through the single pair of functions $\omega_0(x)$ and $\omega_{s_0}(x)$, as extremal solutions of the Kolmogorov-Petrovskii-Piskunov (KPP) traveling wave equation on the half-line, through a martingale representation, and as a single explicit series expansion. We also obtain a precise result concerning the tail behavior of $K(\infty)$. In addition, in the regime where $K(\infty) > 0$ almost surely, we show that $u(x,t) := \mathbb{P}^x(K(t) = 0)$ suitably centered converges to the KPP critical travelling wave on the whole real line.

1 Introduction

Consider a branching Brownian motion in which particles move according to a Brownian motion with drift $\mu \in \mathbb{R}$ and split into two particles at rate β independently one from another. Call $\mathcal{N}_{\text{all}}(t)$ the population of all particles at time t and call $X_u(t)$ the position of a given particle $u \in \mathcal{N}_{\text{all}}(t)$. When we start with a single particle at position x we write $\mathbb{P}^x$ for the law of this process.

In a seminal paper, [21], Kesten considered the branching Brownian motion with absorption, that is, the model just described with the additional property that particles entering the negative half-line $(-\infty, 0]$ are immediately absorbed and removed. We write $\mathcal{N}_{\text{live}}(t)$ for the set of particles alive (not absorbed) in the branching Brownian motion with absorption and $K(t)$ the number of particles that have been absorbed up to time t. The system with absorption is said to become extinct if $\exists t \geq 0 : \mathcal{N}_{\text{live}}(t) = \emptyset$ and to survive otherwise. We let $K(\infty) := \lim_{t\to\infty} K(t) \in \mathbb{N}\cup\{\infty\}$.

Depending on the value of μ one has the following behaviours (see Figure 1)

Regime A: if $\mu \leq -\sqrt{2\beta}$, the drift towards origin is so large that the system goes extinct almost surely. $K(\infty)$ is finite and non-zero.

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