Journal of Functional Analysis 273 (2017) 2249–2274

Contents lists available at ScienceDirect

Journal of Functional Analysis

www.elsevier.com/locate/jfa

On the eigenvalues of the infinitesimal generator of a semigroup of composition operators

Dimitrios Betsakos

Department of Mathematics, Aristotle University of Thessaloniki, 54124 Thessaloniki, Greece

ARTICLE INFO

Article history:
Received 6 March 2015
Accepted 27 June 2017
Available online 8 July 2017
Communicated by K. Seip

MSC:
47B33
47D06
30H10
30C45
30C85

Keywords:
Semigroup of composition operators
Eigenvalues
Spectrum
Infinitesimal generator
Domains convex in one direction
Harmonic measure

ABSTRACT

Suppose that (ϕ_t) is a one-parameter semigroup of holomorphic self-maps of the unit disk with associated planar domain Ω. Let (T_t) be the corresponding semigroup of composition operators on the classical Hardy space H^p. When the semigroup (ϕ_t) is hyperbolic, we describe the point spectrum of the infinitesimal generator of (T_t) in terms of the geometry of Ω. The proofs involve various estimates of harmonic measure. We also present various properties of the point spectrum in the parabolic case.

E-mail address: betsakos@math.auth.gr.

http://dx.doi.org/10.1016/j.jfa.2017.06.025

1. Introduction

1.1. Semigroups of holomorphic functions

Let $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ be the unit disk. A one-parameter family $(\phi_t)_{t\geq 0}$ of holomorphic functions $\phi_t : \mathbb{D} \to \mathbb{D}$ is called a semigroup if it satisfies the conditions:

(a) ϕ_0 is the identity in $\mathbb{D}$.
(b) $\phi_{t+s} = \phi_t \circ \phi_s$, for all $t, s \geq 0$.
(c) $\lim_{t\to s} \phi_t(z) = \phi_s(z)$, for all $s \geq 0$ and all $z \in \mathbb{D}$.

Since $(\phi_t)_{t\geq 0}$ is a uniformly bounded family of holomorphic functions, the pointwise convergence in (c) is, in fact, uniform on compact subsets of $\mathbb{D}$.

The modern theory of semigroups of holomorphic functions was initiated by Berkson and Porta [3] in 1978. That paper contains not only the fundamentals of the theory but also results and insights that have influenced much of the subsequent research. By now, the study of semigroups has been developed to a very rich theory with deep and beautiful connections with several other branches of Analysis, including operator theory and geometric function theory. The reader can find the basic results of the theory and various topics of recent research in [1,7,13,17,22–24].

By a theorem of Berkson and Porta [3], if (ϕ_t) is a semigroup of holomorphic mappings on $\mathbb{D}$ then for each $z \in \mathbb{D}$, the function $t \mapsto \phi_t(z)$ is differentiable and there exists a unique holomorphic function $G : \mathbb{D} \to \mathbb{C}$ such that

$$\frac{\partial \phi_t(z)}{\partial t} = G(\phi_t(z)), \quad t \geq 0, \ z \in \mathbb{D}. \tag{1.1}$$

The function G is called the *infinitesimal generator of the semigroup.*

The main dynamical property of a semigroup is the following [3]: Suppose that (ϕ_t) is a semigroup which is not a group of elliptic automorphisms of the unit disk; (recall that an elliptic automorphism of $\mathbb{D}$ is a conformal mapping of $\mathbb{D}$ onto itself with one fixed point in $\mathbb{D}$). Then there exists a point $\tau \in \mathbb{D} \cup \partial\mathbb{D}$ such that

$$\lim_{t\to\infty} \phi_t(z) = \tau \tag{1.2}$$

and the convergence is uniform on compact subsets of $\mathbb{D}$. The point τ is called the *Denjoy–Wolff point* of the semigroup. We will be interested only in the case $\tau \in \partial\mathbb{D}$. In this case, by composing with suitable automorphisms of $\mathbb{D}$, we may assume without essential loss of generality that $\tau = 1$.

If $\tau = 1$ is the Denjoy–Wolff point of (ϕ_t) then each function ϕ_t has angular derivative at 1 and $0 < \phi_t'(1) \leq 1$. Moreover, Siskakis [23, Chapter 1] proved that either $\phi_t'(1) = 1$ for all $t > 0$ or $\phi_t'(1) < 1$ for all $t > 0$. In the former case the semigroup is called *parabolic* and in the latter case it is called *hyperbolic*.