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The Boltzmann equation with weakly inhomogeneous data in bounded domain

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ABSTRACT

This paper is concerned with the Boltzmann equation with specular reflection boundary condition. We construct a unique global solution and obtain its large time asymptotic behavior in the case that the initial data is close enough to a radially symmetric homogeneous datum. The result extends the case of Cauchy problem considered by Arkeryd–Esposito–Pulvirenti [4] to the specular reflection boundary value problem in bounded domain.

Contents

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1. Introduction

1.1. The problem

If a dilute gas is contained in a bounded region with completely smooth boundary surface and the gas molecules collide the surface elastically, the motion of those gas particles can be modeled by the following initial value problem for the Botlzmann equation with specular reflection boundary condition

$$\begin{cases} \partial_t F + v\cdot\nabla_x F = Q(F,F),\ t>0,\ x\in\Omega,\ v\in\mathbb{R}^3, \\ F(0,x,v) = F_0(x,v),\ x\in\Omega,\ v\in\mathbb{R}^3, \\ F(t,x,v)|_{n(x)\cdot v<0} = F(t,x,R_x v),\ R_x v = v - 2(v\cdot n(x))n(x), \\ \quad t\geq 0,\ x\in\Omega,\ v\in\mathbb{R}^3. \end{cases} \tag{1.1}$$

Here, $F(t,x,v)\geq 0$ denotes the density distribution function of the gas particles at time $t\geq 0$, position $x\in\Omega$, and velocity $v\in\mathbb{R}^3$, Ω is a bounded domain in $\mathbb{R}^3$, $n(x)$ is the outward pointing unit norm vector at boundary $x\in\partial\Omega$. The Boltzmann collision operator $Q(\cdot,\cdot)$ for hard sphere model is given as the following non-symmetric form

$$\begin{aligned} Q(F_1,F_2) &= \int\limits_{\mathbb{R}^3\times\mathbb{S}^2} |(u-v)\cdot\omega|[F_1(u')F_2(v') - F_1(u)F_2(v)]du d\omega \\ &= Q_{\text{gain}}(F_1,F_2) - Q_{\text{loss}}(F_1,F_2), \end{aligned}$$

here, (u,v) and (u',v') stand for the velocities of the particles before and after the collision and satisfy

$$\begin{cases} v' = v + [(u-v)\cdot\omega]\omega, \quad u' = u - [(u-v)\cdot\omega]\omega, \\ |u|^2 + |v|^2 = |u'|^2 + |v'|^2. \end{cases}$$

In the present paper, we study the existence of unique global solution of (1.1) and its time asymptotic behaviors when the initial particle distribution $F_0(x,v)$ is a small perturbation around a spatially homogeneous radially symmetric datum. Specifically speaking, let $G_0(v) = G(|v|)$ be a spatially homogeneous datum and set $F_0 = G_0 + f_0$. We hope to construct unique global solution of (1.1) in the case that f_0 is small in a suitable sense.