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Ando dilations, von Neumann inequality, and distinguished varieties

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ABSTRACT

Let $\mathbb{D}$ denote the unit disc in the complex plane $\mathbb{C}$ and let $\mathbb{D}^2 = \mathbb{D} \times \mathbb{D}$ be the unit bidisc in $\mathbb{C}^2$. Let (T_1, T_2) be a pair of commuting contractions on a Hilbert space $\mathcal{H}$. Let $\dim \operatorname{ran}(I_{\mathcal{H}} - T_j T_j^*) < \infty$, $j = 1, 2$, and let T_1 be a pure contraction. Then there exists a variety $V \subseteq \overline{\mathbb{D}}^2$ such that for any polynomial $p \in \mathbb{C}[z_1, z_2]$, the inequality

$$\|p(T_1, T_2)\|_{\mathcal{B}(\mathcal{H})} \leq \|p\|_V$$

holds. If, in addition, T_2 is pure, then

$$V = \{(z_1, z_2) \in \mathbb{D}^2 : \det(\Psi(z_1) - z_2 I_{\mathbb{C}^n}) = 0\}$$

is a distinguished variety, where Ψ is a matrix-valued analytic function on $\mathbb{D}$ that is unitary-valued on $\partial\mathbb{D}$. Our results comprise a new proof, as well as a generalization, of Agler and McCarthy's sharper von Neumann inequality for pairs of commuting and strictly contractive matrices.

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Notation

$\mathbb{D}$	Open unit disc in the complex plane $\mathbb{C}$.
$\mathbb{D}^2$	Open unit bidisc in $\mathbb{C}^2$.
$\mathcal{H}$, $\mathcal{E}$	Hilbert spaces.
$\mathcal{B}(\mathcal{H})$	The space of all bounded linear operators on $\mathcal{H}$.
$H^2_{\mathcal{E}}(\mathbb{D})$	$\mathcal{E}$-valued Hardy space on $\mathbb{D}$.
M_z	Multiplication operator by the coordinate function z.
$H^\infty_{\mathcal{B}(\mathcal{E})}(\mathbb{D})$	Set of $\mathcal{B}(\mathcal{E})$-valued bounded analytic functions on $\mathbb{D}$.

All Hilbert spaces are assumed to be over the complex numbers. For a closed subspace $\mathcal{S}$ of a Hilbert space $\mathcal{H}$, we denote by $P_{\mathcal{S}}$ the orthogonal projection of $\mathcal{H}$ onto $\mathcal{S}$.

1. Introduction

The famous von Neumann inequality [12] states that: if T is a linear operator on a Hilbert space $\mathcal{H}$ of norm one or less (that is, T is a contraction), then for any polynomial $p \in \mathbb{C}[z]$, the inequality

$$\|p(T)\|_{\mathcal{B}(\mathcal{H})} \leq \|p\|_{\mathbb{D}}$$

holds. Here $\|p\|_{\mathbb{D}}$ denotes the supremum of $|p(z)|$ over the unit disc $\mathbb{D}$.

In 1953, Sz.-Nagy [9] proved that a linear operator on a Hilbert space is a contraction if and only if the operator has a unitary dilation. This immediately gives a simple and elegant proof of the von Neumann inequality.

In 1963, Ando [3] proved the following generalization of Sz.-Nagy's dilation theorem: Any pair of commuting contractions has a commuting unitary dilation. As an immediate consequence, we obtain the following two variables von Neumann inequality:

Theorem *(Ando). Let (T_1, T_2) be a pair of commuting contractions on a Hilbert space $\mathcal{H}$. Then for any polynomial $p \in \mathbb{C}[z_1, z_2]$, the inequality*

$$\|p(T_1, T_2)\|_{\mathcal{B}(\mathcal{H})} \leq \|p\|_{\mathbb{D}^2}$$

holds.

However, for three or more commuting contractions the above von Neumann type inequality is not true in general (see [4,11]). An excellent source of further information on von Neumann inequality is the monograph by Pisier [7].

In a recent seminal paper, Agler and McCarthy [2] proved a sharper version of von Neumann inequality for pairs of commuting and strictly contractive matrices (see Theorem 3.1 in [2]): Two variables von Neumann inequality can be improved in the case of