Journal of Functional Analysis 272 (2017) 1104–1120

Contents lists available at ScienceDirect

Journal of Functional Analysis

www.elsevier.com/locate/jfa

Quasidiagonal traces on exact C^*-algebras

James Gabe

Department of Mathematics and Computer Science, The University of Southern Denmark, Campusvej 55, DK-5230 Odense M, Denmark

A R T I C L E   I N F O

Article history:
Received 6 February 2016
Accepted 23 October 2016
Available online 3 November 2016
Communicated by Stefaan Vaes

MSC:
46L05
46L35
46L80

Keywords:
Traces
Nuclear maps
Order zero maps
Quasidiagonal C^*-algebras

A B S T R A C T

Recently, it was proved by Tikuisis, White and Winter that any faithful trace on a separable, nuclear C^*-algebra in the UCT class is quasidiagonal. Building on their work, we generalise the result, and show that any faithful, amenable trace on a separable, exact C^*-algebra in the UCT class is quasidiagonal. We also prove that any amenable trace on a separable, exact, quasidiagonal C^*-algebra in the UCT class is quasidiagonal.

1. Introduction

The classification of separable, nuclear C^*-algebras using K-theory and traces, the Elliott classification programme, has seen prominent progress over last 25 years. The most recent success shows that separable, unital, simple, non-elementary C^*-algebras in the UCT class with finite nuclear dimension are classified by their Elliott invariant. This very complete classification result builds on a long line of results over the last 25 years, such as [15,19,18], and recent papers of (Elliott), Gong, Lin and Niu [12,9], where the

E-mail address: jamiegabe123@hotmail.com.

http://dx.doi.org/10.1016/j.jfa.2016.10.022
0022-1236/

result was proved under the additional assumption that all traces on the C^*-algebras were quasidiagonal. Soon after, it was shown by Tikuisis, White and Winter in [24], that any trace on a simple, nuclear C^*-algebra in the UCT class is quasidiagonal, thus implying the above result.

One of the early major successes in the classification programme was the Kirchberg–Phillips theorem [15,19], the classification of separable, nuclear, unital, simple, purely infinite C^*-algebras in the UCT class, by the K-groups and the position of the unit in K_0. The UCT class refers to the class of separable C^*-algebras satisfying the universal coefficient theorem of Rosenberg and Schochet [22], or equivalently, the class of separable C^*-algebras which are KK-equivalent to abelian C^*-algebras.

In Kirchberg's approach to the classification result (see [15] or [21]), much more is actually proved. It follows from this approach, that if A is a separable, exact, unital C^*-algebra in the UCT class and B is a unital, purely infinite C^*-algebra, then any pointed homomorphism $\phi\colon (K_*(A), [1_A]_0) \to (K_*(B), [1_B]_0)$ lifts to a full, unital, *nuclear* $*$-homomorphism $A \to B$. Using KK-theory (resp. total K-theory) one even obtains uniqueness results for such nuclear $*$-homomorphisms up to asymptotic (resp. approximate) unitary equivalence.

The same idea was employed by Dadarlat in [8]. He showed that if A and B are separable, simple, unital, tracially AF C^*-algebras, and A is exact and in the UCT class, then any pointed, order preserving homomorphism $\phi\colon (K_*^+(A), [1_A]_0) \to (K_*^+(B), [1_B]_0)$ lifts to a nuclear $*$-homomorphism $A \to B$. He also obtains uniqueness of such $*$-homomorphisms up to approximate unitary equivalence using total K-theory. This reproves the classification of separable, nuclear, unital, simple tracially AF C^*-algebras in the UCT class by Lin [18].

It turns out that this phenomenon often occurs. Rather than considering nuclear C^*-algebras, one might as well consider nuclear maps for which the domain is an exact C^*-algebra. This actually also explains why classification only holds for nuclear C^*-algebras: by using the methods for nuclear $*$-homomorphisms with exact domains, the isomorphism one would construct by the classification results would have to be nuclear. Thus the C^*-algebras we classify would have to be nuclear by the characterisation of nuclear C^*-algebras as the C^*-algebras for which the identity map is nuclear, due to Choi and Effros [6], and Kirchberg [14].

Popa's work on simple quasidiagonal C^*-algebras in [20] was the main inspiration for Lin's definition of tracially AF C^*-algebras, which is a key component in the classification programme. Quasidiagonality of traces was later introduced by Brown in [2] and has also proved to play an important part in the classification programme. Very recently, Tikuisis, White and Winter showed in [24] that any faithful trace on a separable, nuclear C^*-algebra in the UCT class is quasidiagonal. This result has several remarkable consequences. It follows that (1): the Blackadar–Kirchberg problem [1, Question 9.1] has an affirmative answer for simple C^*-algebras in the UCT class, (2): the Rosenberg conjecture is true (see appendix of [13]), and as mentioned above (3): separable, unital, simple, non-elementary C^*-algebras in the UCT class with finite nuclear dimension are