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Bubbling solutions for a skew-symmetric Chern–Simons system in a torus

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ABSTRACT

We establish the existence of bubbling solutions for the following skew-symmetric Chern–Simons system

$$\begin{cases} \Delta u_1 + \dfrac{1}{\varepsilon^2} e^{u_2}(1 - e^{u_1}) = 4\pi \displaystyle\sum_{i=1}^{N_1} \delta_{p_i^1} \\ \Delta u_2 + \dfrac{1}{\varepsilon^2} e^{u_1}(1 - e^{u_2}) = 4\pi \displaystyle\sum_{i=1}^{N_2} \delta_{p_i^2} \end{cases}$$

over a parallelogram Ω with doubly periodic boundary condition, where $\varepsilon > 0$ is a coupling parameter, and δ_p denotes the Dirac measure concentrated at p. We obtain that if $(N_1 - 1)(N_2 - 1) > 1$, there exists an $\varepsilon_0 > 0$ such that, for any $\varepsilon \in (0, \varepsilon_0)$, the above system admits a solution $(u_{1,\varepsilon}, u_{2,\varepsilon})$ satisfying $u_{1,\varepsilon}$ and $u_{2,\varepsilon}$ blow up simultaneously at the point p^*, and

$$\frac{1}{\varepsilon^2} e^{u_{j,\varepsilon}}(1 - e^{u_{i,\varepsilon}}) \to 4\pi N_i \delta_{p^*}, \quad 1 \le i, j \le 2, \quad i \ne j$$

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as $\varepsilon \to 0$, where the location of the point p^* defined by (1.12) satisfies the condition (1.13).

1. Introduction

There are various Chern–Simons theories are developed to explain certain phenomenons in condensed matter physics and particles physics over the past few decades. For example, in condensed matter physics, Chern–Simons terms appear in various anyon models in understanding high temperature superconductivity and fractal quantum Hall effect; in particle physics, Chern–Simons terms allow electrically and magnetically charged vortices. We refer [10] for the survey on the self-dual Chern–Simons theories.

In the works of Hong, Kim and Pac [16], and Jackiw and Weinberg [17], they considered a model with one Chern–Simons gauge field and constructed selfdual Abelian Chern–Simons–Higgs vortices to describe anyonic solitons in $2+1$ dimensions. After [16,17] appeared, Chern–Simons theory with even number gauge fields are suggested in [37,11], in which, the parity invariance can be restored after choosing appropriate coupling constant. This phenomena was observed in the experiments with high temperature superconductors [30].

In this paper, we consider the Chern–Simons model of two Higgs fields, where each of them coupled to one of two Chern–Simons fields. As in [19,9], let $(A_\mu^{(i)})$ $(\mu = 0,1,2,\ i = 1,2)$ be two Abelian gauge fields and ϕ_i $(i = 1,2)$ be two Higgs scalar fields, where the electromagnetic fields and covariant derivatives are defined by

$$F_{\mu\nu}^{(i)} = \partial_\mu A_\nu^{(i)} - \partial_\nu A_\mu^{(i)}, \quad D_\mu \phi_i = \partial_\mu \phi_i - \mathrm{i} A_\mu^{(i)} \phi_i, \quad \mu = 0,1,2,\ i = 1,2. \tag{1.1}$$

The Lagrangian of this model is defined by

$$\mathcal{L} = -\frac{\varepsilon}{2}\epsilon^{\mu\nu\alpha}\left(A_\mu^{(1)} F_{\mu\nu}^{(2)} + A_\mu^{(2)} F_{\mu\nu}^{(1)}\right) + \sum_{i=1}^{2} D_\mu \phi_i \overline{D^\mu \phi_i} - V(\phi_1,\phi_2), \tag{1.2}$$

where $\varepsilon > 0$ is a coupling parameter, and the Higgs potential $V(\phi_1,\phi_2)$ is taken as

$$V(\phi_1,\phi_2) = \frac{1}{4\varepsilon^2}\left(|\phi_2|^2\left[|\phi_1|^2 - 1\right]^2 + |\phi_1|^2\left[|\phi_2|^2 - 1\right]^2\right). \tag{1.3}$$

After a BPS reduction [2,29], the energy minimizer is shown to satisfy the following self-dual equation: