ARTICLE IN PRESS YJFAN:7761

Journal of Functional Analysis ••• (••••) •••–•••

Contents lists available at ScienceDirect

Journal of Functional Analysis

www.elsevier.com/locate/jfa

Generator masas in q-deformed Araki–Woods von Neumann algebras and factoriality

Panchugopal Bikram [a,*], Kunal Mukherjee [b]

[a] School of Mathematical Sciences, National Institute of Science Education and Research Bhubaneswar, HBNI, Jatni, Odisha-752050, India
[b] Department of Mathematics, IIT Madras, Chennai, 600036, India

ARTICLE INFO

Article history:
Received 17 January 2017
Accepted 5 March 2017
Available online xxxx
Communicated by Stefaan Vaes

MSC:
primary 46L10
secondary 46L65, 46L55

Keywords:
q-commutation relations
Von Neumann algebras
MASA

ABSTRACT

To any strongly continuous orthogonal representation of $\mathbb{R}$ on a real Hilbert space $\mathcal{H}_{\mathbb{R}}$, Hiai constructed q-deformed Araki–Woods von Neumann algebras for $-1 < q < 1$, which are W^*-algebras arising from non-tracial representations of the q-commutation relations, the latter yielding an interpolation between the Bosonic and Fermionic statistics. We prove that if the orthogonal representation is not ergodic then these von Neumann algebras are factors whenever $dim(\mathcal{H}_{\mathbb{R}}) \geq 2$ and $q \in (-1, 1)$. In such case, the centralizer of the q-quasi free state has trivial relative commutant. In the process, we study 'generator MASAs' in these factors and establish that they are strongly mixing.

1. Introduction

In free probability, Voiculescu's C^*-free Gaussian functor associates a canonical C^*-algebra denoted by $\Gamma(\mathcal{H}_{\mathbb{R}})$ to a real Hilbert space $\mathcal{H}_{\mathbb{R}}$, the former being generated by $s(\xi)$, $\xi \in \mathcal{H}_{\mathbb{R}}$, where each $s(\xi)$ is the sum of creation and annihilation operators on

* Corresponding author.
E-mail addresses: bikram@niser.ac.in (P. Bikram), kunal@iitm.ac.in (K. Mukherjee).

http://dx.doi.org/10.1016/j.jfa.2017.03.005
0022-1236/© 2017 Published by Elsevier Inc.

Please cite this article in press as: P. Bikram, K. Mukherjee, Generator masas in q-deformed Araki–Woods von Neumann algebras and factoriality, J. Funct. Anal. (2017), http://dx.doi.org/10.1016/j.jfa.2017.03.005

the full Fock space of the complexification of $\mathcal{H}_{\mathbb{R}}$. The associated von Neumann algebra $\Gamma(\mathcal{H}_{\mathbb{R}})''$ is isomorphic to $L(\mathbb{F}_{dim(\mathcal{H}_{\mathbb{R}})})$ and is the central object in the study of free probability (see [39] for more on the subject). In the literature, there are three interesting types of deformations of Voiculescu's free Gaussian functor each of which has a real Hilbert space $\mathcal{H}_{\mathbb{R}}$ as the initial input data: (i) the q-Gaussian functor due to Bożejko and Speicher for $-1 < q < 1$ (see [4]), (ii) a functor due to Shlyakhtenko (see [31]) which is a free probability analog of the construction of quasi free states on the CAR and CCR algebras and (iii) the third one is a combination of the first two and is due to Hiai (see [21]); the associated von Neumann algebras are respectively called Bożejko–Speicher factors (or q-Gaussian von Neumann algebras), free Araki–Woods factors and q-deformed Araki–Woods von Neumann algebras.

Historically, for the first time, Frisch and Bourret in [15] had considered operators satisfying the q-canonical commutation relations:

$$l(e)l(f)^* - ql(f)^*l(e) = \langle e, f\rangle I, -1 < q < 1.$$

The existence of such operators on an 'appropriate Fock space' was proved by Bożejko and Speicher in [4] and these operators have importance in particle statistics [16,17]. Since then many experts have studied the q-Gaussian von Neumann algebras. Structural properties of the q-Gaussian algebras have been studied in [1,4,3,5,10,30,27,35,32,33]. A short summary of the results obtained in these studies are as follows. For $dim(\mathcal{H}_{\mathbb{R}}) \geq 2$, the q-Gaussian von Neumann algebras $\Gamma_q(\mathcal{H}_{\mathbb{R}})$ are non-injective, solid, strongly solid, non Γ factors with w^*-completely contractive approximation property. Further, $\Gamma_q(\mathcal{H}_{\mathbb{R}}) \cong L(\mathbb{F}_{dim(\mathcal{H}_{\mathbb{R}})})$ for values of q sufficiently close to zero [18].

The Shlyakhtenko functor in [31] associates a C^*-algebra $\Gamma(\mathcal{H}_{\mathbb{R}}, U_t)$ to a pair $(\mathcal{H}_{\mathbb{R}}, U_t)$, where $\mathcal{H}_{\mathbb{R}}$ is a real Hilbert space and (U_t) is a strongly continuous real orthogonal representation of $\mathbb{R}$ on $\mathcal{H}_{\mathbb{R}}$. The von Neumann algebras $\Gamma(\mathcal{H}_{\mathbb{R}}, U_t)''$ obtained this way i.e., the free Araki–Woods von Neumann algebras are full factors of type III_λ, $0 < \lambda \leq 1$, when (U_t) is non-trivial and $dim(\mathcal{H}_{\mathbb{R}}) \geq 2$ [31]. These von Neumann algebras are type III counterparts of the free group factors. In short, they satisfy the complete metric approximation property, lack Cartan subalgebras, are strongly solid, and, they satisfy Connes' bicentralizer problem when they are type III_1 (see [22,23,2]). They have many more interesting properties.

The third functor mentioned above is the q-deformed functor due to Hiai for $-1 < q < 1$ (see [21]). Hiai's functor is the main topic of this paper. It is a combination of Bożejko–Speicher's functor and Shlyakhtenko's functor. This functor, like the Shlyakhtenko's functor, associates a C^*-algebra $\Gamma_q(\mathcal{H}_{\mathbb{R}}, U_t)$ to a pair $(\mathcal{H}_{\mathbb{R}}, U_t)$, where $\mathcal{H}_{\mathbb{R}}$ is a real Hilbert space and (U_t) is a strongly continuous orthogonal representation of $\mathbb{R}$ on $\mathcal{H}_{\mathbb{R}}$ as before. The associated von Neumann algebras in this construction a priori depend on $q \in (-1, 1)$ and are represented in standard form on 'twisted full Fock spaces' that carry the spectral data of (U_t) and connects it to the modular theory of this particular standard representation in a manner such that the canonical creation and annihilation operators satisfy