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Reflection positive doubles

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ABSTRACT

Here we introduce *reflection positive doubles*, a general framework for reflection positivity, covering a wide variety of systems in statistical physics and quantum field theory. These systems may be bosonic, fermionic, or parafermionic in nature. Within the framework of reflection positive doubles, we give necessary and sufficient conditions for reflection positivity. We use a reflection-invariant cone to implement our construction. Our characterization allows for a direct interpretation in terms of coupling constants, making it easy to check in concrete situations. We illustrate our methods with numerous examples.

Contents

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1. Introduction

There is amazing synergy among a number of developments in operator algebra theory, quantum field theory, and statistical physics that first emerged in the 1960's and 1970's. At the time several of these advances appeared independently, but we now understand them as part of a larger picture. Their interrelation may well lead to further deep insights.

The advances we think of include, on the side of mathematics, the Tomita–Takesaki theory for von Neumann algebras [42,41], the j-positive states of Woronowicz [43], and the self-dual cones of Araki, Connes, and Haagerup [1,4,18]. On the side of physics, they