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Calderón–Zygmund estimates for a class of quasilinear elliptic equations

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In this paper we obtain the following local Calderón–Zygmund estimates

$$B(|\mathbf{f}|) \in L^q_{loc}(\Omega) \Rightarrow B(|\nabla u|) \in L^q_{loc}(\Omega) \quad \text{for any} \quad q \geq 1$$

of weak solutions for a class of quasilinear elliptic equations

$$\operatorname{div}(a(|\nabla u|)\nabla u) = \operatorname{div}(a(|\mathbf{f}|)\mathbf{f}) \quad \text{in} \quad \Omega,$$

where $B(t) = \int_0^t \tau a(\tau)\, d\tau$ for $t \geq 0$.

1. Introduction

In this paper we are concerned with the local L^p-type regularity estimates of weak solutions for the following quasilinear elliptic equations

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$$\operatorname{div}\left(a\left(|\nabla u|\right)\nabla u\right)=\operatorname{div}\left(a\left(|\mathbf{f}|\right)\mathbf{f}\right) \quad \text{in } \Omega, \tag{1.1}$$

where Ω is an open bounded domain in $\mathbb{R}^n$, the function $a:(0,\infty)\to(0,\infty)\in C^1\left((0,\infty)\right)$ and satisfies

$$-1<i_a=:\inf_{t>0}\frac{ta'(t)}{a(t)}\leq\sup_{t>0}\frac{ta'(t)}{a(t)}=:s_a<\infty. \tag{1.2}$$

Especially when $a(t)=t^{p-2}$, (1.1) is reduced to p-Laplace equation

$$\operatorname{div}\left(|\nabla u|^{p-2}\nabla u\right)=\operatorname{div}\left(|\mathbf{f}|^{p-2}\mathbf{f}\right) \quad \text{in } \Omega \tag{1.3}$$

and condition (1.2) is exactly $1<p<+\infty$.

L^p-type regularity is the fundamental theory for elliptic and parabolic equations, which plays an important role in the theory of partial differential equations, and is the basis for the existence and uniqueness of solutions. DiBenedetto and Manfredi [14], and Iwaniec [16] obtained L^q $(q\geq p)$ estimates of the gradient of weak solutions for (1.3) when $p>1$. Their results were extended by many authors (see [2,4,5,11–13,17–20]) with different assumptions on the coefficients and domains. Recently, Cianchi and Maz'ya [8] proved global Lipschitz regularity for the Dirichlet and Neumann elliptic boundary value problems of the form

$$\operatorname{div}\left(a\left(|\nabla u|\right)\nabla u\right)=f \quad \text{in } \Omega \tag{1.4}$$

with the condition (1.2). Moreover, they [9] also obtained the corresponding result for the system case. Furthermore, Cianchi and Maz'ya [10] obtained a sharp estimate for the decreasing rearrangement of the length of the gradient for the Dirichlet and Neumann elliptic boundary value problems of (1.4) with (1.2),

$$Ct^{p-1}\leq ta(t)\leq C(t^{p-1}+1) \quad \text{for any } t>0 \text{ and } p\in[2,n)$$

and weak assumptions on the boundary.

The main purpose of this paper is to establish a local Calderón–Zygmund theory

$$B\left(|\mathbf{f}|\right)\in L^q_{loc}(\Omega)\Rightarrow B\left(|\nabla u|\right)\in L^q_{loc}(\Omega) \quad \text{for any } \ q\geq 1 \tag{1.5}$$

for a local weak solution of (1.1), where

$$B(t)=\int_0^t \tau a(\tau)\,d\tau \quad \text{for } \ t\geq 0.$$

Indeed, if $a(t)=t^{p-2}$, (1.5) is reduced to the classical L^q estimates for the p-Laplace equation (1.3)