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Long time asymptotics of non-symmetric random walks on crystal lattices

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A B S T R A C T

In the present paper, we study long time asymptotics of non-symmetric random walks on crystal lattices from a view point of discrete geometric analysis due to Kotani and Sunada [11,25]. We observe that the Euclidean metric associated with the standard realization of the crystal lattice, called the Albanese metric, naturally appears in the asymptotics. In the former half of the present paper, we establish two kinds of (functional) central limit theorems for random walks. We first show that the Brownian motion on the Euclidean space with the Albanese metric appears as the scaling limit of the usual central limit theorem for the random walk. Next we introduce a family of random walks which interpolates between the original non-symmetric random walk and the symmetrized one. We then capture the Brownian motion with a constant drift of the asymptotic direction on the Euclidean space with the Albanese metric associated with the symmetrized random walk through another kind of central limit theorem for the family of random walks. In the latter half of the present paper, we give a spectral geometric proof of the asymptotic expansion of the n-step transition probability for the non-symmetric random walk. This asymptotic expansion is a refinement of the local central limit theorem obtained by Sunada [22,23] and

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is a generalization of the result in [11] for symmetric random walks on crystal lattices to non-symmetric cases.
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1. Introduction

Let $X = (V, E)$ be an oriented, locally finite connected graph (which may have multiple edges and loops). For an oriented edge $e \in E$, the origin and the terminus of e are denoted by $o(e)$ and $t(e)$, respectively. The inverse edge of $e \in E$ is denoted by $\overline{e}$. Let $E_x = \{e \in E |\ o(e) = x\}$ be the set of edges with $o(e) = x \in V$. A path c of X of length n is a sequence $c = (e_1, \dots, e_n)$ of oriented edges e_i with $o(e_{i+1}) = t(e_i)$ $(i = 1, \dots, n-1)$. We denote by $\Omega_{x,n}(X)$ $(x \in V, n \in \mathbb{N} \cup \{\infty\})$ the set of all paths of length n for which origin $o(c) = x$. For simplicity, we also write $\Omega_x(X) := \Omega_{x,\infty}(X)$.

A random walk on X is a stochastic process with values in X characterized effectively by a transition probability, a non-negative function $p : E \to \mathbb{R}$ satisfying

$$\sum_{e \in E_x} p(e) = 1 \quad (x \in V), \qquad p(e) + p(\overline{e}) > 0 \quad (e \in E), \tag{1.1}$$

where $p(e)$ stands for the probability that a particle at $o(e)$ moves to $t(e)$ along the edge e in one unit time. The transition operator L on X associated with the random walk is defined by

$$Lf(x) := \sum_{e \in E_x} p(e) f(t(e)) \qquad (x \in V).$$

The n-step transition probability $p(n, x, y)$ $(n \in \mathbb{N},\ x, y \in V)$ is defined by

$$p(n, x, y) := \sum_{c=(e_1,\dots,e_n)} p(e_1) \cdots p(e_n), \tag{1.2}$$

where the sum is taken over all paths $c = (e_1, \dots, e_n)$ of length n with the origin $o(c) = x$ and the terminus $t(c) = y$. We mention

$$L^n f(x) = \sum_{y \in V} p(n, x, y) f(y) \qquad (x \in V).$$

In a natural manner, the transition probability p induces the probability measure $\mathbb{P}_x$ on the set $\Omega_x(X)$. The random walk associated with p is the time homogeneous Markov chain $(\Omega_x(X), \mathbb{P}_x, \{w_n\}_{n=0}^{\infty})$ with values in X defined by

$$w_n(c) := o(c(n+1)) \qquad (n = 0, 1, 2, \dots,\ c \in \Omega_x(X)),$$

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