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A rich structure related to the construction of analytic matrix functions ☆

D.C. Brown [a], Z.A. Lykova [a], N.J. Young [b,a,*]

[a] *School of Mathematics and Statistics, Newcastle University, Newcastle upon Tyne, NE1 7RU, UK*
[b] *School of Mathematics, Leeds University, Leeds LS2 9JT, UK*

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We study certain interpolation problems for analytic 2×2 matrix-valued functions on the unit disc. We obtain a new solvability criterion for one such problem, a special case of the μ-synthesis problem from robust control theory. For certain domains $\mathcal{X}$ in $\mathbb{C}^2$ and $\mathcal{C}^3$ we describe a rich structure of interconnections between four objects: the set of analytic functions from the disc into $\mathcal{X}$, the 2×2 matricial Schur class, the Schur class of the bidisc, and the set of pairs of positive kernels on the bidisc subject to a boundedness condition. This rich structure combines with the classical realisation formula and Hilbert space models in the sense of Agler to give an effective method for the construction of the required interpolating functions.

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* Corresponding author.
E-mail addresses: d.c.brown@newcastle.ac.uk (D.C. Brown), Zinaida.Lykova@ncl.ac.uk (Z.A. Lykova), N.J.Young@leeds.ac.uk (N.J. Young).

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Contents

1. Introduction

Engineering provides some hard challenges for classical analysis. In signal processing and, in particular, control theory, one often needs to construct analytic matrix-valued functions on the unit disc $\mathbb{D}$ or right half-plane subject to finitely many interpolation conditions and to some subtle boundedness requirements. The resulting problems are close in spirit to the classical Nevanlinna–Pick problem, but established operator- or function-theoretic methods which succeed so elegantly for the classical problem do not seem to help for even minor variants. For example, this is so for the *spectral Nevanlinna–Pick problem* [13,21], which is to construct an analytic square-matrix-valued function F in $\mathbb{D}$ that satisfies a finite collection of interpolation conditions and the boundedness condition

$$\sup_{\lambda\in\mathbb{D}} r(F(\lambda)) \le 1 \quad \text{for all } \lambda \in \mathbb{D}.$$

This problem is a special case of the *μ-synthesis problem* of H^∞ control, which is recognised as a hard and important problem in the theory of robust control [18,19]. Even the special case of the spectral Nevanlinna–Pick problem for 2×2 matrices awaits a definitive analytic theory.

A major difficulty in μ-synthesis problems is to describe the analytic maps from $\mathbb{D}$ to a suitable domain $\mathcal{X} \subset \mathbb{C}^n$ or its closure $\overline{\mathcal{X}}$. In the classical theory $\mathcal{X}$ is a matrix ball, and the *realisation formula* presents the general analytic map from $\mathbb{D}$ to $\mathcal{X}$ in terms of a contractive operator on Hilbert space; this formula provides a powerful approach