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A conjecture for the regularized fourth moment of Eisenstein series

Goran Djanković [a,1], Rizwanur Khan [b,*]

[a] *University of Belgrade, Faculty of Mathematics, Studentski Trg 16, p.p. 550, 11000 Belgrade, Serbia*
[b] *Texas A&M University at Qatar, PO Box 23874, Doha, Qatar*

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ABSTRACT

We formulate a version of the Random Wave Conjecture for the fourth moment of Eisenstein series which is based on Zagier's regularized inner product. We prove an asymptotic formula expressing the regularized fourth moment as a mean value of L-functions. This is an advantage over previous work in the literature, which has approached the fourth moment problem through truncated Eisenstein series and not yielded a suitable expression in terms of L-functions.

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* Corresponding author.
E-mail addresses: djankovic@matf.bg.ac.rs (G. Djanković), rizwanur.khan@qatar.tamu.edu (R. Khan).
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1. Introduction

One of the main research themes in recent years in the theory of automorphic forms is the problem of mass distribution. Let $X = \Gamma\backslash\mathbb{H}$, where $\mathbb{H}$ is the upper half complex plane and $\Gamma = SL_2(\mathbb{Z})$. In his PhD thesis, Spinu [Sp] obtained the following type of weak equidistribution result:

$$\int\limits_X |E_A(z, \frac{1}{2} + iT)|^4 d\mu z \ll T^\epsilon, \tag{1.1}$$

where $d\mu(z) = \frac{dxdy}{y^2}$ and $E_A(z, s)$ is the truncated Eisenstein series, which on the fundamental domain equals $E(z, s)$ for $\mathrm{Im}(z) \leq A$, and $E(z, s)$ minus its constant term for $\mathrm{Im}(z) > A$. See the next section for a more careful definition. Spinu's result (see also [Lu] for a closely related result) is in line with a much more general conjecture, called the Random Wave Conjecture. This conjecture was made for Eisenstein series in [HR, section 7.3]. In terms of moments this implies: for any even integer $p \geq 0$ and any nice compact $\Omega \subset X$, we should have

$$\lim_{T\to\infty} \frac{1}{\mathrm{vol}(\Omega)} \int\limits_\Omega \left|\frac{E_A(z, \frac{1}{2} + iT)}{\sqrt{2\log T}}\right|^p d\mu z = \frac{c_p}{\mathrm{vol}(X)^{p/2}}, \tag{1.2}$$

where c_p is the pth moment of the normal distribution $\mathcal{N}(0, 1)$. The same conjecture is also made for $E(z, \frac{1}{2}+iT)$. As we will see below, $\sqrt{2\log T}$ roughly equals $\|E_A(\cdot, \frac{1}{2}+iT)\|_2$.

One would of course like to go beyond Spinu's upper bound and prove an asymptotic for the fourth moment of Eisenstein series. In [BK], this was achieved, conditional on the Generalized Lindelöf Hypothesis, for Hecke Maass forms of large eigenvalue when $\Omega = X$, and agreement was found with the RWC. Thus in analogy one would expect (1.2) to also hold for $p = 4$ and $\Omega = X$, and one may hope that the statement in this case can be proven unconditionally. After all, such problems can be a bit easier for Eisenstein series – for example, recall that the case $p = 2$ of (1.2) was first proven for Eisenstein series [LS] before the analogue was proven for Hecke Maass forms [Li,So].

What would the proof of such an asymptotic entail? The starting point in [BK] is to relate the fourth moment of an L^2-normalized Hecke Maass form f to L-functions. One uses the spectral decomposition and Plancherel's theorem to write

$$\langle f^2, f^2\rangle = \sum_{j\geq 1} |\langle f^2, u_j\rangle|^2 + \dots, \tag{1.3}$$

where the inner product is the Petersson inner product, $\{u_j : j \geq 1\}$ is an orthonormal basis of Hecke Maass forms, and the ellipsis denotes the contribution of the Eisenstein spectrum and constant eigenfunction. Next one can use Watson's triple product formula to relate the squares of the inner products on the right hand side to central values of L-functions. Thus the problem is reduced to one of obtaining a mean value of L-functions.

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