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## SUMS OF TWO RATIONAL CUBES WITH MANY PRIME FACTORS

DONGHO BYEON AND KEUNYOUNG JEONG

**Abstract.** In this paper, we show that for any given integer  $k \geq 2$ , there are infinitely many cube-free integers  $n$  having exactly  $k$  prime divisors such that  $n$  is a sum of two rational cubes. This is a cubic analogue of the work of Tian [Ti], which proves that there are infinitely many congruent numbers having exactly  $k$  prime divisors for any given integer  $k \geq 1$ .

### 1. INTRODUCTION AND RESULTS

Let  $n$  be a cube-free integer and  $E_n : x^3 + y^3 = n$  the elliptic curve defined over  $\mathbb{Q}$ . Let  $L_{E_n}(s)$  be the Hasse-Weil  $L$ -function of  $E_n$  and  $w_n \in \{1, -1\}$  its root number. Then  $L_{E_n}(s)$  satisfies the functional equation

$$N^{s/2}(2\pi)^{-s}\Gamma(s)L_{E_n}(s) = w_n N^{(2-s)/2}(2\pi)^{-(2-s)}\Gamma(2-s)L_{E_n}(2-s),$$

where  $N$  is the conductor of  $E_n$  whose divisors are 3 and primes  $p \mid n$ . The *analytic rank* of  $E_n$  is the order of vanishing at the central point  $s = 1$  of  $L_{E_n}(s)$ . The functional equation implies that  $w_n = 1$  if and only if the analytic rank of  $E_n$  is even. The Birch and Swinnerton-Dyer(BSD) conjecture states that the rank of the Mordell-Weil group  $E_n(\mathbb{Q})$  is equal to the analytic rank of  $E_n$ . So the BSD conjecture implies that if  $w_n = -1$ , then  $n$  is a sum of two rational cubes.

The root number  $w_n$  can be computed by the following way, due to Birch and Stephens [BS],

$$w_n = \prod_{p \text{ prime}} w_n(p),$$

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