

# Accepted Manuscript

Some identities for multiple Hurwitz zeta values

Zhongyan Shen, Lirui Jia

PII: S0022-314X(17)30151-8  
DOI: <http://dx.doi.org/10.1016/j.jnt.2017.03.003>  
Reference: YJNTH 5724

To appear in: *Journal of Number Theory*

Received date: 17 August 2016  
Revised date: 9 March 2017  
Accepted date: 11 March 2017

Please cite this article in press as: Z. Shen, L. Jia, Some identities for multiple Hurwitz zeta values, *J. Number Theory* (2017), <http://dx.doi.org/10.1016/j.jnt.2017.03.003>

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.



# Some identities for multiple Hurwitz zeta values

Zhongyan Shen\*, Lirui Jia\*\*

*Department of Mathematics, Zhejiang International Studies University, Hangzhou 310012, P.R. China*

*Department of Mathematics, Zhejiang University, Hangzhou 310027, P.R. China*

*\*E-mail address: huanchenszhan@163.com*

*\*\*E-mail address: jialirui@126.com*

**Abstract** For any positive integers  $k, m, n$  with  $m \geq 2$  and  $k \leq n$ , let  $T(m, n, k)$  be the sum of all the multiple Hurwitz zeta values

$$\zeta\left(s_1, s_2, \dots, s_k; -\frac{1}{2}, -\frac{1}{2}, \dots, -\frac{1}{2}\right)$$

of weight  $mn$  and depth  $k$  with arguments being multiples of  $m$ , i.e.,

$$T(m, n, k) = \sum_{\substack{s_1 + s_2 + \dots + s_k = n \\ s_j \in \mathbb{N}}} \zeta\left(ms_1, ms_2, \dots, ms_k; -\frac{1}{2}, -\frac{1}{2}, \dots, -\frac{1}{2}\right).$$

In this note, we obtain the evaluation

$$T(m, n, k) = \sum_{\substack{a+b=n \\ a, b \in \mathbb{N}_0}} (-1)^{a-k} \binom{a}{k} \cdot \zeta\left(\{m; -\frac{1}{2}\}^a\right) \cdot \zeta^*\left(\{m; -\frac{1}{2}\}^b\right),$$

where  $\zeta\left(\{m; -\frac{1}{2}\}^a\right)$  and  $\zeta^*\left(\{m; -\frac{1}{2}\}^b\right)$  denote the multiple Hurwitz zeta values and multiple Hurwitz zeta-star values respectively. Moreover, we give the evaluations of  $\zeta\left(\{m; -\frac{1}{2}\}^n\right)$  and  $\zeta^*\left(\{m; -\frac{1}{2}\}^n\right)$  when  $m$  is even.

**Keywords** multiple Hurwitz zeta values, multiple Hurwitz zeta-star values, generating function

**MSC** primary 11M32, 11M35; secondary 11B68

## 1 Introduction and main results

The purpose of this paper is to give some identities for multiple Hurwitz zeta values. The paper is organized as follows. In this section, we recall multiple zeta values and their sum formulas, multiple Hurwitz zeta values and give the main results for multiple Hurwitz zeta values. In section 2, we cite an extended version of Lemma 2.1 [6] that is very useful for proving the main results. Finally, in section 3, we prove the main results.

---

This work is supported by the Natural Science Foundation of Zhejiang Province, Project (No. LQ13A010012) and the National Natural Science Foundation of China, Project (No. 11571303).

Download English Version:

<https://daneshyari.com/en/article/5772515>

Download Persian Version:

<https://daneshyari.com/article/5772515>

[Daneshyari.com](https://daneshyari.com)