

Accepted Manuscript

Reflection principles for class groups

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PII: S0022-314X(17)30073-2
DOI: <http://dx.doi.org/10.1016/j.jnt.2017.01.008>
Reference: YJNTH 5682

To appear in: *Journal of Number Theory*

Received date: 11 July 2016
Revised date: 15 January 2017
Accepted date: 15 January 2017

Please cite this article in press as: J. Klys, Reflection principles for class groups, *J. Number Theory* (2017), <http://dx.doi.org/10.1016/j.jnt.2017.01.008>

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REFLECTION PRINCIPLES FOR CLASS GROUPS

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ABSTRACT. We present several new examples of reflection principles which apply to both class groups of number fields and picard groups of curves over $\mathbb{P}^1/\mathbb{F}_p$. This proves a conjecture of Lemmermeyer [3] about equality of 2-rank in subfields of A_4 , up to a constant not depending on the discriminant in the number field case, and exactly in the function field case. More generally we prove similar relations for subfields of a Galois extension with group G for the cases when G is S_3 , S_4 , A_4 , D_{2l} and $\mathbb{Z}/l\mathbb{Z} \rtimes \mathbb{Z}/r\mathbb{Z}$. The method of proof uses sheaf cohomology on 1-dimensional schemes, which reduces to Galois module computations.

INTRODUCTION

In this paper we look at the problem of relating the size of the l -torsion in the class groups of two distinct number fields. We let $\mathrm{rk}_l Cl(K) = \dim_{\mathbb{F}_l} Cl(K)[l]$.

Describing the size of the l -torsion of the class group of a number field is in general a hard problem. There are special cases where one can say something about $\mathrm{rk}_l Cl(K)$. For example it is easy to show for any quadratic field K that $\mathrm{rk}_2 Cl(K) = r - 1$, where r is the number of primes ramified in K . One source of theorems describing l -torsion in class groups is Iwasawa theory, which gives formulas for $\mathrm{rk}_l Cl(K)$ for K lying in some tower of number fields, in terms of certain invariants depending on the base field. A very strong asymptotic conjecture on class group order is the following due to Zhang [8]:

Conjecture. *For any number field K let $n = [K : \mathbb{Q}]$ and let $\epsilon > 0$. Then*

$$|Cl(K)[l]| \ll_{\epsilon, n, l} D_K^\epsilon.$$

That is the l -torsion in number fields of a fixed degree grows slower than any power of their discriminant. Some work has been done in this direction by Ellenberg and Venkatesh [2] by combining reflection principles with analytic techniques.

A slightly different kind of problem is that of relating l -rank in class groups of two different number fields. Such statements are often called reflection principles. There are many such statements known and we will give a very brief overview of some of these. For a more exhaustive exposition we refer the reader to [3].

The following is referred to as the Scholz reflection theorem [7]:

Proposition 1. *Let $D > 1$ be square-free. Let $K = \mathbb{Q}(\sqrt{-3D})$ and $F = \mathbb{Q}(\sqrt{D})$. Then $\mathrm{rk}_l Cl(F) \leq \mathrm{rk}_l Cl(K) \leq \mathrm{rk}_l Cl(F) + 1$.*

There is the following special case of Leopoldt's theorem, originally due to Hecke [7]:

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