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# ON TERNARY PROBLEMS IN ADDITIVE PRIME NUMBER THEORY

LILU ZHAO

ABSTRACT. We consider the quadratic exponential sums

$$f_D(\alpha) = \sum_{d \leq D} \left| \sum_{P/d < x \leq 2P/d} e(d^2 x^2 \alpha) \right|.$$

It is established that in some average sense, one has  $f_D(\alpha) \ll P^{1/2+\varepsilon} D^{3/4}$ . As applications, we improve two results concerning ternary problems in additive prime number theory.

## 1. INTRODUCTION

The philosophy of the Hardy-Littlewood circle method suggests that we may expect to solve the equation

$$n = x_1^{k_1} + x_2^{k_2} + \cdots + x_s^{k_s},$$

in natural numbers  $x_j (1 \leq j \leq s)$ , where the exponents  $k_j (1 \leq j \leq s)$  are fixed integers satisfying

$$\sum_{j=1}^s \frac{1}{k_j} > 1$$

and  $n$  is sufficiently large satisfying the condition that the congruence

$$x_1^{k_1} + x_2^{k_2} + \cdots + x_s^{k_s} \equiv n \pmod{q}$$

has solutions for all  $q$ . In this paper, we consider the ternary additive equations

$$n = x^2 + p_1^3 + p_2^k \tag{1.1}$$

with multiplicative restrictions on the variable  $x$ . Throughout, the letter  $p$ , with or without a subscript, will denote a prime number. In order to capture multiplicative restrictions on the variable  $x$  in (1.1), we are lead to study the following quadratic exponential sums

$$f_D(\alpha) = \sum_{d \leq D} \left| \sum_{P/d < x \leq 2P/d} e(d^2 x^2 \alpha) \right|. \tag{1.2}$$

The argument in the proof of Lemma 4.2 of Brüdern and Kawada [2] implies

$$f_D(\alpha) \ll P^{1/2+\varepsilon} D \tag{1.3}$$

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