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ON TERNARY PROBLEMS IN ADDITIVE PRIME NUMBER THEORY

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ABSTRACT. We consider the quadratic exponential sums

$$f_D(\alpha)=\sum_{d\leqslant D}\Big|\sum_{P/d<x\leqslant 2P/d}e(d^2x^2\alpha)\Big|.$$

It is established that in some average sense, one has $f_D(\alpha)\ll P^{1/2+\varepsilon}D^{3/4}$. As applications, we improve two results concerning ternary problems in additive prime number theory.

1. INTRODUCTION

The philosophy of the Hardy-Littlewood circle method suggests that we may expect to solve the equation

$$n=x_1^{k_1}+x_2^{k_2}+\cdots+x_s^{k_s},$$

in natural numbers $x_j(1\leqslant j\leqslant s)$, where the exponents $k_j(1\leqslant j\leqslant s)$ are fixed integers satisfying

$$\sum_{j=1}^{s}\frac{1}{k_j}>1$$

and n is sufficiently large satisfying the condition that the congruence

$$x_1^{k_1}+x_2^{k_2}+\cdots+x_s^{k_s}\equiv n(\mathrm{mod}\ q)$$

has solutions for all q. In this paper, we consider the ternary additive equations

$$n=x^2+p_1^3+p_2^k \tag{1.1}$$

with multiplicative restrictions on the variable x. Throughout, the letter p, with or without a subscript, will denote a prime number. In order to capture multiplicative restrictions on the variable x in (1.1), we are lead to study the following quadratic exponential sums

$$f_D(\alpha)=\sum_{d\leqslant D}\Big|\sum_{P/d<x\leqslant 2P/d}e(d^2x^2\alpha)\Big|. \tag{1.2}$$

The argument in the proof of Lemma 4.2 of Brüdern and Kawada [2] implies

$$f_D(\alpha)\ll P^{1/2+\varepsilon}D \tag{1.3}$$

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