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Bing-Ling Wu, Yong-Gao Chen

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# On certain properties of harmonic numbers

Bing-Ling Wu<sup>a</sup>, Yong-Gao Chen<sup>a,\*</sup>

<sup>a</sup>*School of Mathematical Sciences and Institute of Mathematics  
Nanjing Normal University, Nanjing 210023, P R China*

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## Abstract

Let  $H_n$  be the  $n$ -th harmonic number and let  $u_n$  be its numerator. For any prime  $p$ , let  $J_p$  be the set of positive integers  $n$  with  $p \mid u_n$ . In 1991, Eswarathasan and Levine conjectured that  $J_p$  is finite for any prime  $p$ . It is clear that the  $p$ -adic valuation of  $H_n$  is not less than  $-\lfloor \log_p n \rfloor$ . Let  $T_p$  be the set of positive integer  $n$  such that the  $p$ -adic valuation of  $H_n$  is equal to  $-\lfloor \log_p n \rfloor$ . Recently, Carlo Sanna proved that  $|J_p \cap [1, x]| \leq 129p^{2/3}x^{0.765}$  and that there exists  $S_p \subseteq T_p$  with  $\delta(S_p) > 0.273$ , where  $\delta(X)$  denotes the logarithmic density of the set  $X$  of positive integers. He also commented that with his methods  $\delta(S_p) > 1/3 - \varepsilon$  cannot be achieved. In this paper, we improve these results. For example, two of our results are: (a)  $|J_p \cap [1, x]| \leq 3x^{2/3+1/(25 \log p)}$ ; (b)  $\delta(T_p)$  exists and  $1 - (2 \log p)^{-1} \leq \delta(T_p) \leq 1 - (p \log p)^{-1}$  for all primes  $p \geq 13$ . In particular,  $\delta(T_p) > 0.63$  for all primes  $p$ .

*Keywords:* Harmonic numbers;  $p$ -adic valuation; Asymptotic density; Logarithmic density.

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## 1. Introduction

For any positive integer  $n$ , let

$$H_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}.$$

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\*Correspond author

*Email addresses:* 390712592@qq.com ( Bing-Ling Wu), ygchen@njnu.edu.cn (Yong-Gao Chen)

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