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Abstract

In this paper, we define some new continued fraction sequences towards Euler's constant and two related inequalities. We also present some numerical simulations to demonstrate the superiority of the optimal new sequences over new sequences with other coefficients and Lu's sequences at the end of this article.

Key words: Euler's constant; continued fraction; rate of convergence; asymptotic expansion; inequalities.
2000 MSC: 11Y60; 11A55; 41A25; 34E05; 33F05.

1. Introduction

As it is known to us, defining some new approximations towards fundamental constants plays an important role in field of mathematical constants. One of the most famous constants is Euler's constant, which is denoted as

$$\gamma = 0.577215\ldots.$$

And we also know that sequence γ_n is defined as

$$\gamma_n = \sum_{k=1}^{n} \frac{1}{k} - \ln n, \tag{1.1}$$

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