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NEW LOWER BOUNDS FOR THE LEAST COMMON MULTIPLE OF POLYNOMIAL SEQUENCES

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ABSTRACT. Let n be a positive integer and $f(x)$ be a polynomial with nonnegative integer coefficients. We prove that $\mathrm{lcm}_{\lceil n/2\rceil\le i\le n}\{f(i)\}\ge 2^{n-1}\sqrt{\lceil n/2\rceil}$ for any positive integer n, where $\lceil n/2\rceil$ denotes the smallest integer that is not less than $n/2$. This improves the lower bound obtained by Hong, Luo, Qian and Wang in 2013. For the least common multiple of the first n positive integers, we show that $\mathrm{lcm}_{1\le i\le n}\{i\}\ge 2^{n-3}(n-1)\sqrt{\frac{n-2}{2}}$ for any integer $n\ge 7$, which improves the lower bound obtained by Nair in 1982 and by Farhi in 2009. For the least common multiple of consecutive quadratic progression terms, by using the integration method combined with a little more detailed analysis on the absolute value of complex numbers, we further show that $\mathrm{lcm}_{\lceil n/2\rceil\le i\le n}\{ai^2+c\}\ge 2^{n-1}\sqrt{\lceil n/2\rceil}\cdot\min(a,\sqrt{ac})$ for any positive integer n, where a and c are two positive integers. This improves and extends the results obtained by Farhi in 2005 and Oon in 2013, respectively.

1. **Introduction**

The least common multiple of consecutive positive integers was first investigated by Chebyshev [2], who made an important progress for the determination of the order of magnitude of the prime counting function and also show that the prime number theorem is equivalent to the statement: $\log\mathrm{lcm}(1,...,n)\sim n$ as n tends to infinity. Since then, the least common multiple of sequences of integers received a lot of attention from many authors. Some authors studied the asymptotic behavior of the least common multiple of integer sequence. The readers can refer to [1, 14, 20]. Some authors also explored the smallest period problems associated with the least common multiple of polynomial sequences (see, for example, [3, 4, 6, 12, 13, 16, 21]).

Effective bounds for the least common multiple of integer sequences are also given by several authors. Hanson [7] proved that $\mathrm{lcm}_{1\le i\le n}\{i\}<3^n$ for any integer $n\ge 1$. Nair [18] showed by a remarkably simple and effective method that $\mathrm{lcm}_{1\le i\le n}\{i\}\ge 2^n$ for any integer $n\ge 7$ (see also [22]). Farhi [5] proved an interesting identity involving the least common multiple of binomial coefficients and obtained $\mathrm{lcm}_{1\le i\le n}\{i\}\ge n\binom{n-1}{\lfloor\frac{n-1}{2}\rfloor}$ as an application. Lower bounds of the least common multiple of finite arithmetic progression are investigated by Farhi [3, 4], Hong and Feng [9], Hong and Yang [15], Hong and Kominers [10], Kane and Kominers [17] and Wu, Tan and Hong [24]. For the quadratic case, some results are also achieved. Farhi [3, 4] provided a nontrivial lower bound for

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