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Lattice map for Anderson t-motives: First approach ☆

A. Grishkov [a,b], D. Logachev [c,*]

[a] *Departamento de Matemática e Estatistica, Universidade de São Paulo, Rua de Matão 1010, CEP 05508-090, São Paulo, Brazil*
[b] *Omsk State University n.a. F.M.Dostoevskii, Pr. Mira 55-A, Omsk 644077, Russia*
[c] *Departamento de Matemática, Universidade Federal do Amazonas, Manaus, Brazil*

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There exists a lattice map from the set of pure uniformizable Anderson t-motives to the set of lattices. It is not known what is the image and the fibers of this map. We prove a local result that sheds the first light to this problem and suggests that maybe this map is close to 1–1. Namely, let $M(0)$ be a t-motive of dimension n and rank $r = 2n$ — the n-th power of the Carlitz module of rank 2, and let M be a t-motive which is in some sense "close" to $M(0)$. We consider the lattice map $M \mapsto L(M)$, where $L(M)$ is a lattice in $\mathbb{C}_\infty^n$. We show that the lattice map is an isomorphism in a "neighborhood" of $M(0)$. Namely, we compare the action of monodromy groups: (a) from the set of equations defining t-motives to the set of t-motives themselves, and (b) from the set of Siegel matrices to the set of lattices. The result of the present paper gives that the size of a neighborhood, where we have an isomorphism, depends on an element of the monodromy group. We do not

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* Corresponding author.
E-mail addresses: shuragri@gmail.com (A. Grishkov), logachev94@gmail.com (D. Logachev).

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know whether there exists a universal neighborhood. Method of the proof: explicit solution of an equation describing an isomorphism between two t-motives by a method of successive approximations using a version of the Hensel lemma.

0. Introduction

t-motives ([G], 5.4.2, 5.4.18, 5.4.16) are the functional field analogs of abelian varieties (more exactly, of abelian varieties with multiplication by an imaginary quadratic field, see for example [L1]). For the number field case we have a classical theorem (here and below we consider lattices up to a linear transformation of the ambient space):

Theorem 0.1. *Abelian varieties of dimension g over $\mathbb{C}$ are in 1–1 correspondence with lattices satisfying Riemann condition, of dimension $2g$ in $\mathbb{C}^g$.*

Our knowledge on the functional field analog of this theorem is very poor, and the purpose of the present paper is to get a result towards this analog. Let $\mathbb{C}_\infty$ be the analog of $\mathbb{C}$ in characteristic p (it is a complete algebraically closed field). Throughout all the paper we consider for simplicity only t-motives M over the affine line A^1 such that their nilpotent operators N (see (1.3.1) below for its definition) are equal to 0. Let r, n be respectively the rank and dimension of M. For some M it is possible associate to M a lattice $L(M)$ of rank r in n-dimensional space $\mathbb{C}_\infty^n$ (see (1.4) for a definition of a lattice). These M are called uniformizable ([A], Section 2). $M \mapsto L(M)$ is a contravariant functor.

For $n = 1$ the situation is completely analogous to the Theorem 0.1:

Theorem 0.2 *([Dr]). All t-motives of dimension 1 (= Drinfeld modules) are uniformizable. There is a 1–1 correspondence between Drinfeld modules of rank r over $\mathbb{C}_\infty$ and lattices of rank r in $\mathbb{C}_\infty$.*

There exists a notion of purity of M (see [G], 5.5.2 for the definition; all Drinfeld modules are pure). For $n = r - 1$ the duality theory gives us an immediate corollary of Theorem 0.2:

Corollary 0.3 *([L], Corollary 8.4). All pure t-motives of rank r and dimension $r-1$ over $\mathbb{C}_\infty$ are uniformizable. There is a 1–1 correspondence between their set, and the set of lattices of rank r in $\mathbb{C}_\infty^{r-1}$ having dual.*

Not all such lattices have dual, but almost all, i.e. even in this simple case the correspondence is not strictly 1–1, but only an "almost 1–1".