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Additive complements of the Squares

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Abstract

Two infinite sequences A and B of nonnegative integers are called *additive complements*, if their sum contains all sufficiently large integers. We also say that B is an additive complement of A if A and B are additive complements. In this paper, we consider a problem of Ben Green on additive complements of the squares: $S = \{1^2, 2^2, \dots\}$. The following result is proved: if $B = \{b_n\}_{n=1}^{\infty}$ with $b_n \ge \frac{\pi^2}{16}n^2 - 0.57n^{\frac{1}{2}}\log n - \beta n^{\frac{1}{2}}$ for all positive integers n and any given constant β, then B is not an additive complement of S. In particular, $B = \left\{ \left\lfloor \frac{\pi^2}{16}n^2 \right\rfloor \mid n = 1, 2, \dots \right\}$ is not an additive complement of S.

Keywords: additive complements, square, counting function
2010 MSC: 11B13, 11B75

1. Introduction

Two infinite sequences A and B of nonnegative integers are called *additive complements*, if their sum contains all sufficiently large integers. We also say that B is an additive complement of A if A and B are additive complements. Let $R_{A,B}(n)$ be the number of solutions of $n = a+b$, $a \in A$, $b \in B$. Hence, if A and B are called additive complements, then $R_{A,B}(n) \ge 1$ for all sufficiently large integers n. Let $\lfloor x \rfloor$ be the integral part of x and let $|A|$ be the cardinality of A. Given an integer $N > 1$. Two subsets A, B of $\{0, 1, \dots, N\}$ are called *additive complements* on $\{0, 1, \dots, N\}$ if every integer n with $0 \le n \le N$ can

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