

# Accepted Manuscript

Additive complements of the Squares

Yong-Gao Chen, Jin-Hui Fang

PII: S0022-314X(17)30202-0  
DOI: <http://dx.doi.org/10.1016/j.jnt.2017.04.016>  
Reference: YJNTH 5765

To appear in: *Journal of Number Theory*

Received date: 26 January 2017  
Revised date: 28 April 2017  
Accepted date: 28 April 2017

Please cite this article in press as: Y.-G. Chen, J.-H. Fang, Additive complements of the Squares, *J. Number Theory* (2017), <http://dx.doi.org/10.1016/j.jnt.2017.04.016>

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.



# Additive complements of the Squares

Yong-Gao Chen<sup>a,\*</sup>, Jin-Hui Fang<sup>b</sup>

<sup>a</sup>*School of Mathematical Sciences and Institute of Mathematics,  
Nanjing Normal University, Nanjing 210023, P R China*

<sup>b</sup>*Department of Mathematics, Nanjing University of Information Science & Technology,  
Nanjing 210044, P R China*

---

## Abstract

Two infinite sequences  $A$  and  $B$  of nonnegative integers are called *additive complements*, if their sum contains all sufficiently large integers. We also say that  $B$  is an additive complement of  $A$  if  $A$  and  $B$  are additive complements. In this paper, we consider a problem of Ben Green on additive complements of the squares:  $S = \{1^2, 2^2, \dots\}$ . The following result is proved: if  $B = \{b_n\}_{n=1}^{\infty}$  with  $b_n \geq \frac{\pi^2}{16}n^2 - 0.57n^{\frac{1}{2}} \log n - \beta n^{\frac{1}{2}}$  for all positive integers  $n$  and any given constant  $\beta$ , then  $B$  is not an additive complement of  $S$ . In particular,  $B = \left\{ \left\lfloor \frac{\pi^2}{16}n^2 \right\rfloor \mid n = 1, 2, \dots \right\}$  is not an additive complement of  $S$ .

*Keywords:* additive complements, square, counting function

*2010 MSC:* 11B13, 11B75

---

## 1. Introduction

Two infinite sequences  $A$  and  $B$  of nonnegative integers are called *additive complements*, if their sum contains all sufficiently large integers. We also say that  $B$  is an additive complement of  $A$  if  $A$  and  $B$  are additive complements. Let  $R_{A,B}(n)$  be the number of solutions of  $n = a+b$ ,  $a \in A$ ,  $b \in B$ . Hence, if  $A$  and  $B$  are called additive complements, then  $R_{A,B}(n) \geq 1$  for all sufficiently large integers  $n$ . Let  $\lfloor x \rfloor$  be the integral part of  $x$  and let  $|A|$  be the cardinality of  $A$ . Given an integer  $N > 1$ . Two subsets  $A, B$  of  $\{0, 1, \dots, N\}$  are called *additive complements on  $\{0, 1, \dots, N\}$*  if every integer  $n$  with  $0 \leq n \leq N$  can

---

\*Corresponding author

*Email addresses:* [ygchen@njnu.edu.cn](mailto:ygchen@njnu.edu.cn) (Yong-Gao Chen),  
[fangjinhui1114@163.com](mailto:fangjinhui1114@163.com) ( Jin-Hui Fang)

Download English Version:

<https://daneshyari.com/en/article/5772616>

Download Persian Version:

<https://daneshyari.com/article/5772616>

[Daneshyari.com](https://daneshyari.com)