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Davenport–Hasse's Theorem for Polynomial Gauss Sums over Finite Fields

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Abstract

In this paper, we study the polynomial Gauss sums over finite fields, and present an analogue of Davenport–Hasse's theorem for the polynomial Gauss sums, which is a generalization of the previous result obtained by Hayes.

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1 Introduction

Let $\mathbb{F}_q$ be a finite field with $q = p^l$ elements, where p is a prime number. Let $\mathbb{F}_{q^n}$ be a finite extension over $\mathbb{F}_q$ of degree n, and σ be the Frobinus on $\mathbb{F}_{q^n}$, given by $\sigma(a) = a^q$ for any elements a in $\mathbb{F}_{q^n}$. We have $\sigma^n = 1$, and σ generates the Galois group of $\mathbb{F}_{q^n}/\mathbb{F}_q$. The relative trace $\mathrm{tr}(a)$ and the norm $\mathrm{N}(a)$ of an element a in $\mathbb{F}_{q^n}$ are defined by

$$\mathrm{tr}(a) = \sum_{i=1}^{n} \sigma^i(a), \quad \mathrm{N}(a) = \prod_{i=1}^{n} \sigma^i(a) \tag{1.1}$$

respectively. Let ψ be a (complex-valued) character of the additive group of $\mathbb{F}_q$, and χ a character of the multiplicative group $\mathbb{F}_q^*$ of $\mathbb{F}_q$. The Gauss sums on $\mathbb{F}_q$ are defined by

$$\tau(\chi, \psi) = \sum_{a \in \mathbb{F}_q^*} \chi(a)\psi(a). \tag{1.2}$$

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