

# Accepted Manuscript

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PII: S0022-314X(17)30209-3  
DOI: <http://dx.doi.org/10.1016/j.jnt.2017.05.007>  
Reference: YJNTH 5772

To appear in: *Journal of Number Theory*

Received date: 1 February 2017  
Revised date: 5 May 2017  
Accepted date: 6 May 2017

Please cite this article in press as: C. Miguel, Cesàro's formula in number fields, *J. Number Theory* (2017), <http://dx.doi.org/10.1016/j.jnt.2017.05.007>

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## Cesàro's formula in number fields

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### Abstract

We give an extension of a theorem of Cesàro from the rational integers to the ring of integers of an arbitrary number field. This extension is used to generalize Pillai's function to number fields.

*Keywords:* Dedekind zeta function, Euler totient function, Gcd-sum function, Number field, Pillai function

*2010 MSC:* 11R42, 11R99

### 1. Introduction

A theorem of Cesàro (see e.g. [5], [8, p.127], [17]) states that for every natural number  $n \in \mathbb{N} = \{1, 2, \dots\}$  and any arithmetical function  $f$  we have

$$\sum_{i=1}^n f((i, n)) = \sum_{d|n} f(d)\varphi(n/d), \quad (1)$$

where  $(i, n)$  denotes the greatest common divisor of  $i$  and  $n$ , and  $\varphi$  is the Euler totient function. There is in the literature a large number of generalizations and analogues of Cesàro theorem. For a rich and extensive survey concerning generalizations of Cesàro theorem the reader is referred to [10]. Since rings of integers in a number field are natural generalizations of the rational integers, the question natural arises as to whether an analogous statement could be made for the ring of integers in a number field. Recently, there has been considerable interest in extending arithmetical identities from the rational integers to a more general setting (see e.g. [11], [13]).

The aim of this paper is to extend Cesàro theorem to the ring  $\mathcal{O}_K$  of integers in a number field  $K$ . Instead of working with elements in  $\mathcal{O}_K$ , where unique factorization can fail, we will work with ideals. As is well known using ideals in place of elements we can save unique factorization. More precisely, every nonzero ideal  $\mathfrak{n}$  of  $\mathcal{O}_K$  can be uniquely written in the form  $\mathfrak{n} = \mathfrak{p}_1^{\alpha_1} \dots \mathfrak{p}_s^{\alpha_s}$ , where  $\mathfrak{p}_1, \dots, \mathfrak{p}_s$  are distinct nonzero prime ideals and  $\alpha_1, \dots, \alpha_s$  are positive rational integers (see e.g. [14, p.8]).

Unique factorization of ideals in  $\mathcal{O}_K$  permits calculations that are analogous to some familiar manipulations involving ordinary integers. In particular we can define the concept of arithmetical function on the set of ideals of  $\mathcal{O}_K$ . A real or complex-valued function defined on the set of ideals of the ring of integers in a number field is called an arithmetical function. As a very simple example, consider a nonzero ideal  $\mathfrak{n}$  of  $\mathcal{O}_K$ , then the generalized Euler totient function, which is denoted by  $\varphi_K(\mathfrak{n})$ , is defined to be the order of the multiplicative group of units in the factor ring  $\mathcal{O}_K/\mathfrak{n}$ , denoted by  $U(\mathcal{O}_K/\mathfrak{n})$ , with the convention that  $\varphi_K(\mathcal{O}_K) = 1$ . That is,

$$\varphi_K(\mathfrak{n}) = \begin{cases} 1 & \text{if } \mathfrak{n} = \mathcal{O}_K, \\ |U(\mathcal{O}_K/\mathfrak{n})| & \text{otherwise.} \end{cases}$$

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